1. Introduction

In 1982, Mashhour et al [9] introduced the notion of pre-open sets and the pre-closed sets were defined in [4]. Ashish Kar and Bhattacharya [2], in 1990, continued their work on pre-open sets and offered another set of separation axioms analogous to the semi separation axioms defined by Maheshwari and Prasad [8]. Caldas [10] defined a new class of sets called semi-Difference (briefly sD) sets by using the semi-open sets [5], and introduced the semi-D$_i$ spaces for $i = 0, 1, 2$.

The purpose of this paper is to introduce some pre-separation axioms in bitopological spaces. To define and investigate the axioms we use the $(1, 2)^*$ pre-open sets [6] introduced by Lellis Thivagar and Ravi. We call these axioms as $(1, 2)^*_{pre-T_0}$, $(1, 2)^*_{pre-T_1}$ and $(1, 2)^*_{pre-T_2}$. We also define $(1, 2)^*_{pre-D_i}$ sets and utilize them to define the $(1, 2)^*_{pre-D_i}$, $i = 0, 1, 2$, axioms. We prove a bitopological space is $(1, 2)^*_{pre-D_1}$ if and only if it is $(1, 2)^*_{pre-D_2}$.

We recall some definitions and concepts which are useful in the following sections.

2. Preliminaries

In this section unless it is explicitly stated $X$ is a topological space $(X, \tau)$. For a $A \subset X$, the interior and closure of $A$ in $X$ are denoted by $int(A)$ and $cl(A)$ respectively.

Definition 2.1. A space $X$ is called

(i). $Pre-T_0$ [2] iff to each pair of distinct points $x, y$ in $X$, there exists a pre-open set containing one of the points but not the other.

(ii). $Pre-T_1$ [2] iff to each pair of distinct points $x, y$ of $X$, there exists a pair of pre-open sets one containing $x$ but not $y$ and other containing $y$ but not $x$.

(iii). $Pre-T_2$ [2] iff to each pair of distinct points $x, y$ of $X$, there exists a pair of disjoint pre-open sets one containing $x$ and the other containing $y$.

Definition 2.2. A subset $A$ of $X$ is called a semi-Difference set (in short sD-set) if there are two semi-open sets $O_1, O_2$ in $X$ such that $O_1 \neq X$ and $A = O_1 \setminus O_2$.

Definition 2.3. A space $X$ is called

(i). Semi-$D_0$ if for $x, y \in X$, $x \neq y$, there exists a sD-set of $X$ containing one of $x$ and $y$ but not the other.

(ii). Semi-$D_1$ if for $x, y \in X$, $x \neq y$, there exists a pair of sD-sets one containing $x$ but not $y$ and the other containing $y$ but not $x$. 

\textbf{Abstract.} It is the object of this paper to introduce the $(1, 2)^*_{pre-D_k}$ axioms for $k = 0, 1, 2$. 

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(iii). Semi-$D_2$ if for $x, y \in X, x \neq y$, there exist disjoint $sD$-sets $S_1$ and $S_2$ such that $x \in S_1$ and $y \in S_2$.

A subset $A$ of a bitopological space $(X, \tau_1, \tau_2)$ is called $\tau_{1,2}$-open [6] if $A = S_1 \cup S_2$ where $S_1 \in \tau_1$ and $S_2 \in \tau_2$, and $\tau_{1,2}$-closed if $A^c$ is $\tau_{1,2}$-open in $X$. We write $\tau_1\tau_2$-interior of $A$ and $\tau_1\tau_2$-closure of $A$, in short form, $\tau_1\tau_2-int(A)$ and $\tau_1\tau_2-cl(A)$ respectively. $\tau_1\tau_2-int(A)$ is the union of all $\tau_{1,2}$-open sets contained in $A$ and $\tau_1\tau_2-cl(A)$ is the intersection of all $\tau_{1,2}$-closed sets containing $A$.

**Definition 2.4.** A subset $A$ of $X$ is called $(1,2)^\ast$-pre-open [6] if $A \subset \tau_1\tau_2-int(\tau_1\tau_2-cl(A))$ and $(1,2)^\ast$-pre-closed if its complement in $X$ is $(1,2)^\ast$-pre-open. Or equivalently, $\tau_1\tau_2-cl(\tau_1\tau_2-int(A)) \subset A$.

The family of all $(1,2)^\ast$-pre-open sets of $X$ is denoted by $(1,2)^\ast PO(X)$. $(1,2)^\ast$-pre-closure of $A$ denoted by $(1,2)^\ast pcl(A)$ is the intersection of all $(1,2)^\ast$-pre-closed sets containing $A$.

A subset $A$ of $X$ is $(1,2)^\ast$-pre-closed if and only if $(1,2)^\ast pcl(A) = A$. Note that every $\tau_{1,2}$-open set is $(1,2)^\ast$-pre-open.

**Definition 2.5.** A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called

(i). $(1,2)^\ast$ pre-irresolute if the inverse image of every $(1,2)^\ast$-pre-open set in $Y$ is $(1,2)^\ast$-pre-open in $X$.

(ii). Strongly $(1,2)^\ast$-pre-open [6] if the image of every $(1,2)^\ast$-pre-open if the image of every $(1,2)^\ast$-pre-open set in $X$ is $(1,2)^\ast$-pre-open in $Y$.

In the next section we introduce the $(1,2)^\ast$-pre-$T_k$-spaces for $k = 0, 1, 2$. In the following sections by $X$ and $Y$ we mean bitopological spaces $(X, \tau_1, \tau_2)$ and $(Y, \sigma_1, \sigma_2)$ respectively.

3. $(1,2)^\ast$-PRE-$T_k$-SPACES

**Definition 3.1.** A bitopological space $X$ is said to be $(1,2)^\ast$-pre-$T_0$ iff for $x, y \in X$, $x \neq y$, there exists an $(1,2)^\ast$-pre-open set containing only one of $x$ and $y$ but not the other.

Now we proceed to prove that every bitopological space is $(1,2)^\ast$-pre-$T_0$.

**Lemma 3.2.** If for some $x \in X$, $\{x\}$ is $(1,2)^\ast$-pre-open then $x \notin (1,2)^\ast pcl(\{y\})$ for all $y \neq x$.

**Proof.** If $\{x\}$ is $(1,2)^\ast$-pre-open for some $x \in X$, then $X \setminus \{x\}$ is $(1,2)^\ast$-pre-closed and $x \notin X \setminus \{x\}$. If $x \in (1,2)^\ast pcl(\{y\})$ for some $y \neq x$, then $x, y$ both are in all the $(1,2)^\ast$-pre-closed sets containing $y$ which implies that $x \in X \setminus \{x\}$ which is not true. Therefore, $x \notin (1,2)^\ast pcl(\{y\})$. \qed

**Theorem 3.3.** In a space $X$, distinct points have distinct $(1,2)^\ast$-pre-closures.

**Proof.** Let $x, y \in X, x \neq y$. Take $A = \{x\}^c$. Then $\tau_1\tau_2-cl(A) = A$ or $X$.

Case (a). If $\tau_1\tau_2-cl(A) = A$, then $A$ is $\tau_{1,2}$-closed and hence $(1,2)^\ast$-pre-closed. Then $X \setminus A = \{x\}$ is $(1,2)^\ast$-pre-open, not containing $y$. Therefore, by Lemma 3.2, $x \notin (1,2)^\ast pcl(\{y\})$ and $y \in (1,2)^\ast pcl(\{y\})$ which implies that $(1,2)^\ast pcl(\{x\})$ and $(1,2)^\ast pcl(\{y\})$ are distinct.

Case (b). If $\tau_1\tau_2-cl(A) = X$, then $A$ is $(1,2)^\ast$-pre-open and hence $\{x\}$ is $(1,2)^\ast$-pre-closed which shows that $(1,2)^\ast pcl(\{x\}) = \{x\}$ which is not equal to $(1,2)^\ast pcl(\{y\})$. \qed
Theorem 3.4. In a space X, if distinct points have distinct (1,2)*pre-closures then X is (1,2)*pre-$T_0$.

Proof. Let $x, y \in X, x \neq y$. Then $(1,2)^*pcl(\{x\})$ is not equal to $(1,2)^*pcl(\{y\})$. Then there exists $z \in X$ such that $z \in (1,2)^*pcl(\{x\})$ but $z \notin (1,2)^*pcl(\{y\})$ or $z \in (1,2)^*pcl(\{y\})$ but $z \notin (1,2)^*pcl(\{x\})$. Without loss of generality, let $z \in (1,2)^*pcl(\{x\})$ but $z \notin (1,2)^*pcl(\{y\})$. If $x \in (1,2)^*pcl(\{y\})$, then $(1,2)^*pcl(\{x\})$ is contained in $(1,2)^*pcl(\{y\})$ and therefore, $z \in (1,2)^*pcl(\{y\})$, which is a contradiction. Thus we get $x \notin (1,2)^*pcl(\{y\})$. This implies that $x \in (1,2)^*pcl((\{y\})^c)$. Therefore, X is (1,2)*pre-$T_0$. □

Theorem 3.5. Every bitopological space is (1,2)*pre-$T_0$.

Proof. Follows from Theorem 3.3 and Theorem 3.4. □

Remark 3.6. It is observed that every $T_0$-space is pre-$T_0$ but not the converse [2]. Here we note that if a space X is $T_0$ with respect to $\tau_1$ or $\tau_2$, then X is (1,2)*pre-$T_0$. But if X is (1,2)*pre-$T_0$, it is not necessary that (X, $\tau_1$) is $T_0$ or (X, $\tau_2$) is $T_0$, as shown in the following example.

Example 3.7. Let $X = \{a, b, c\}$. $\tau_1 = \{\emptyset, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{b, c\}, X\}$. Then X is (1,2)*pre-$T_0$ but both (X, $\tau_1$) and (X, $\tau_2$) are $T_0$.

Definition 3.8. A space X is called (1,2)*pre-$T_1$ iff for $x, y \in X, x \neq y$, there exist $U, V \in (1,2)^*PO(X)$ such that $x \in U, y \notin U$ and $y \in V, x \notin V$.

Remark 3.9. It is obvious that every (1,2)*pre- $T_1$ space is (1,2)*pre-$T_0$ but the converse is not true in general as illustrated in the next example.

Example 3.10. Let $X = \{a, b, c\}$. $\tau_1 = \{\emptyset, \{a\}, X\}$, $\tau_2 = \{\emptyset, \{b\}, X\}$. Then X is (1,2)*pre-$T_0$ but not (1,2)*pre-$T_1$.

Theorem 3.11. In a space X, the following statements are equivalent.

(i). X is (1,2)*pre-$T_1$.

(ii). For each $x \in X$, $\{x\}$ is (1,2)*pre-closed in X.

(iii). Each subset of X is the intersection of all (1,2)*pre-open sets containing it.

(iv). The intersection of all (1,2)*pre-open sets containing the point $x \in X$ is $\{x\}$.

Proof. (i) $\Rightarrow$ (ii).

Let $x \in X$. If $y \in X$ and $x \neq y$ then there exists an (1,2)*pre-open set $U_y$ such that $y \in U_y$. Hence $y \in U_y \subset \{x\}^c$. Therefore, $\{x\}^c = \bigcup \{U_y : y \in \{x\}^c\}$ which is (1,2)*pre-open and so $\{x\}$ is (1,2)*pre-closed in X.

(ii) $\Rightarrow$ (iii).

Let $A \subset X$ and $y \notin A$. Then $A \subset \{y\}^c$ and $\{y\}^c$ is (1,2)*pre-open in X and $A = \bigcap \{y\}^c : y \in A^c$ which is the intersection of all (1,2)*pre-open sets containing A.

(iii) $\Rightarrow$ (iv).

Obvious.

(iv) $\Rightarrow$ (i).

Let $x, y \in X, x \neq y$. By our assumption, there exist at least an (1,2)*pre-open set containing x but not y and also an (1,2)*pre-open set containing y but not x. Therefore, X is (1,2)*pre-$T_1$.

Definition 3.12. A space X is called (1,2)*pre-$T_2$ iff for $x, y \in X, x \neq y$, there exist disjoint (1,2)*pre-open sets $U, V$ in X such that $x \in U$ and $y \in V$.

Remark 3.13. (1,2)*pre-$T_2$ness implies (1,2)*pre-$T_1$ness but the converse is not true in general. In Example 3.7, X is (1,2)*pre-$T_1$ but not (1,2)*pre-$T_2$. 
Definition 3.14. A subset $O$ of $X$ is said to be $(1, 2)^*\text{-pre-neighbourhood of a point } x \in X$ if there exists an $(1, 2)^*\text{-pre-open set } U$ such that $x \in U \subset O$.

Theorem 3.15. For a space $X$ the following statements are equivalent.

(i). $X$ is $(1, 2)^*\text{-pre-T}_2$.

(ii). If $x \in X$, then for each $y \neq x$, there is an $(1, 2)^*\text{-pre-neighbourhood } N(x)$ of $x$ such that $y \notin (1, 2)^*\text{pcl}(N(x))$.

(iii). For each $x \in \{(1, 2)^*\text{pcl}(N) : N$ is an $(1, 2)^*\text{-pre-neighbourhood of } x\}$, $\{x\}$.

Proof. (i) $\Rightarrow$ (ii).

Let $x \in X$. If $y \in X$ is such that $y \neq x$, there exist disjoint $(1, 2)^*\text{-pre-open sets } U, V$ such that $x \in U$ and $y \in V$. Then $x \in U \subset X \setminus V$ which implies that $X \setminus V$ is an $(1, 2)^*\text{-pre-neighbourhood of } x$. Also $X \setminus V$ is $(1, 2)^*\text{-pre-closed and } y \notin X \setminus V$. Let $N(x) = X \setminus V$. Then $y \notin (1, 2)^*\text{pcl}(N(x))$.

(ii) $\Rightarrow$ (iii).

Obvious.

(iii) $\Rightarrow$ (i).

Let $x, y \in X$, $x \neq y$. By hypothesis, there is at least an $(1, 2)^*\text{-pre-neighbourhood } N$ of $x$ such that $y \notin (1, 2)^*\text{pcl}(N)$. We have $x \notin X \setminus (1, 2)^*\text{pcl}(N)$ is $(1, 2)^*\text{-pre-open}$. Since $N$ is an $(1, 2)^*\text{-pre-neighbourhood of } x$, there exists $U \in (1, 2)^*\text{PO}(X)$ such that $x \in U \subset N$ and $U \cap (X \setminus (1, 2)^*\text{pcl}(N)) = \emptyset$. Hence $X$ is $(1, 2)^*\text{-pre-T}_2$. $lacksquare$

Definition 3.16. A space $X$ is said to be $(1, 2)^*\text{-pre-regular}$ if for each $(1, 2)^*\text{-pre-closed set } F$ and each point $x \notin F$ there exist disjoint $(1, 2)^*\text{-pre-open sets } U$ and $V$ such that $x \in U$ and $V \subset F$.

Theorem 3.17. An $(1, 2)^*\text{-pre-T}_0$ space is $(1, 2)^*\text{-pre-T}_2$ if it is $(1, 2)^*\text{-pre-regular}$.

Proof. Let $X$ be $(1, 2)^*\text{-pre-T}_0$ and $(1, 2)^*\text{-pre-regular}$. If $x, y \in X$, $x \neq y$, there exists $U \in (1, 2)^*\text{PO}(X)$ such that $U$ contains one of $x$ and $y$, say $x$ but not $y$. Then $X \setminus U$ is $(1, 2)^*\text{-pre-closed and } x \notin X \setminus U$. Since $X$ is $(1, 2)^*\text{-pre-regular}$, there exist disjoint $(1, 2)^*\text{-pre-open sets } V_1$ and $V_2$ such that $x \in V_1$ and $X \setminus U \subset V_2$. Thus $x \in V_1$ and $y \in V_2$, $V_1 \cap V_2 = \emptyset$. Hence $X$ is $(1, 2)^*\text{-pre-T}_2$. $lacksquare$

Theorem 3.18. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is an injective, $(1, 2)^*\text{-pre-irresolute map and } Y$ is $(1, 2)^*\text{-pre-T}_2$ then $X$ is $(1, 2)^*\text{-pre-T}_2$.

Proof. Let $x, y \in X$, $x \neq y$. Since $f$ is injective, $f(x) \neq f(y)$ in $Y$ and there exist disjoint $(1, 2)^*\text{-pre-open sets } U, V$ such that $f(x) \in U$ and $f(y) \in V$. Let $G = f^{-1}(U)$ and $H = f^{-1}(V)$. Then $x \in G$, $y \in H$ and $G, H \in (1, 2)^*\text{PO}(X)$. Also $G \cap H = f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = \emptyset$. Thus $X$ is $(1, 2)^*\text{-pre-T}_2$.

4. Pre-difference axioms

Definition 4.1. A subset $A$ of $X$ is called $(1, 2)^*\text{-pre-difference set (briefly } (1, 2)^*\text{pD-set})$ if there are two $(1, 2)^*\text{-pre-open sets } P_1$ and $P_2$ in $X$, $P_1 \neq X$ such that $A = P_1 \setminus P_2$.

Remark 4.2. It is evident that each $(1, 2)^*\text{-pre-open set is an } (1, 2)^*\text{pD-set}$.

Now we define another set of separation axioms called $(1, 2)^*\text{-pre-D}_i$, $i = 0, 1, 2$ by using the $(1, 2)^*\text{pD-sets}$.

Definition 4.3. A space $X$ is said to be

(i). $(1, 2)^*\text{-pre-D}_0$ if for $x, y \in X$, $x \neq y$, there exists an $(1, 2)^*\text{pD-set containing one of } x$ and $y$ but not the other.

(ii). $(1, 2)^*\text{-pre-D}_1$ if for $x, y \in X$, $x \neq y$, there exist $(1, 2)^*\text{pD-sets } U, V$ in $X$ such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$. 

Theorem 4.9. \( (1,2)^*\text{pre-}D_2 \) if for \( x, y \in X, x \neq y \), there exist disjoint \((1,2)^*\text{pD-sets}\) \( U, V \) in \( X \) such that \( x \in U \) and \( y \in V \).

Remark 4.4. (i) Every \((1,2)^*\text{pre-}T_i \) space is \((1,2)^*\text{pre-}D_i \), \( i = 0, 1, 2 \) respectively.

(ii) If \( X \) is \((1,2)^*\text{pre-}D_i \) then it is \((1,2)^*\text{pre-}D_{i-1} \), \( i = 1, 2 \).

Remark 4.5. \((1,2)^*\text{pre-}D_i \) ness does not imply \((1,2)^*\text{pre-}T_i \) ness for \( i = 1, 2 \) respectively.

In Example 3.10, \( X \) is \((1,2)^*\text{pre-}D_1 \) but not \((1,2)^*\text{pre-}T_1 \) and in Example 3.7, \( X \) is not \((1,2)^*\text{pre-}T_2 \) but \((1,2)^*\text{pre-}D_2 \).

The next example shows that \((1,2)^*\text{pre-}D_0 \) does not imply \((1,2)^*\text{pre-}D_1 \).

Example 4.6. Let \( X = \{a, b\}, \tau_1 = \{\emptyset, X\}, \tau_2 = \{\emptyset, \{a\}, X\}. \) Then \( X \) is \((1,2)^*\text{pre-}D_0 \) but not \((1,2)^*\text{pre-}D_1 \).

Theorem 4.7. A space \( X \) is \((1,2)^*\text{pre-}D_0 \) if and only if it is \((1,2)^*\text{pre-}T_0 \).

Proof. Suppose that \( X \) is \((1,2)^*\text{pre-}D_0 \). Let \( x, y \in X, x \neq y \). Then there exists an \((1,2)^*\text{pD-set}\) \( U \) such that \( U \) contains \( x \) but not \( y \), say. As \( U \) is an \((1,2)^*\text{pD-set}\), it is possible to write \( U = P_1 \setminus P_2 \) where \( P_1 \neq X \) and \( P_1, P_2 \in (1,2)^*\text{PO}(X) \). Now there arises two cases. (i) \( y \notin P_1 \) (ii) \( y \in P_1 \) and \( y \in P_2 \).

Case (i). \( y \notin P_1 \) and \( x \in P_1 \setminus P_2 \) implies that \( x \in P_1 \) and \( y \notin P_1 \).

Case (ii). \( y \in P_1 \) and \( y \notin P_2 \). \( x \in P_1 \setminus P_2 \) implies that \( x \notin P_2 \). Thus \( y \in P_2 \) and \( x \notin P_2 \).

Thus in both the cases, we obtain that \( X \) is \((1,2)^*\text{pre-}T_0 \). Conversely, if \( X \) is \((1,2)^*\text{pre-}T_0 \), by Remark 4.4, \( X \) is \((1,2)^*\text{pre-}D_0 \). \( \Box \)

It has been showed in section 3, that an \((1,2)^*\text{pre-}T_2 \)-space is \((1,2)^*\text{pre-}T_1 \) but not the converse. But in the case of pre-Difference axioms, we prove that an \((1,2)^*\text{pre-}D_1 \)-space is \((1,2)^*\text{pre-}D_2 \) and so the \((1,2)^*\text{pre-}D_1 \)-space coincides with \((1,2)^*\text{pre-}D_2 \)-space.

Theorem 4.8. A space \( X \) is \((1,2)^*\text{pre-}D_1 \) if and only if \( X \) is \((1,2)^*\text{pre-}D_2 \).

Proof. Necessity. Let \( x, y \in X, x \neq y \). Then there exist \((1,2)^*\text{pD-sets}\) \( U, V \) in \( X \) such that \( x \in U, y \notin U \) and \( y \in V, x \notin V \). Let \( U = P_1 \setminus P_2 \) and \( V = P_3 \setminus P_4 \) where \( P_i \in (1,2)^*\text{PO}(X) \), \( i = 1, 2, 3, 4 \) and \( P_1 \neq X, P_3 \neq X \). It is evident that \( x \notin V \) implies the two possibilities, (i) \( x \in P_3 \cap P_4 \) (ii) \( x \notin P_3 \).

Case (i). \( x \in P_3 \cap P_4 \). We have \( x \in P_4 \) and \( y \in P_3 \setminus P_4 \) and \( P_4 \cap (P_3 \setminus P_4) = \emptyset \) are disjoint.

Case (ii). \( x \notin P_3, y \notin U \) implies that either \( y \in P_1 \) and \( y \in P_2 \) or \( y \notin P_1 \).

Sub Case (a). \( y \in P_1 \) and \( y \in P_2 \) and \( x \notin P_1 \setminus P_2 \). We get \( P_1 \setminus P_2 \) and \( P_2 \) are disjoint \((1,2)^*\text{pD-sets}\) containing \( x \) and \( y \) respectively.

Sub Case (b). \( y \notin P_1 \) and \( x \in P_1 \setminus P_2 \) and \( x \notin P_1 \setminus P_2 \) implies that \( y \in (P_1 \cup P_4) \) and \( y \notin P_1 \) implies that \( x \in (P_1 \cup P_3) \) and \( y \notin P_1 \) implies that \( x \in (P_2 \cup P_3) \) and \( (P_2 \cup P_3) \) are disjoint. Therefore, \( X \) is \((1,2)^*\text{pre-}D_2 \).

Sufficiency. Follows from Remark 4.4. \( \Box \)

Theorem 4.9. If \( X \) is \((1,2)^*\text{pre-}D_1 \) then it is \((1,2)^*\text{pre-}T_0 \).

Proof. Follows from (ii) Remark 4.4 and theorem 4.8.

Remark 4.10. An \((1,2)^*\text{pre-}T_0 \)-space is not \((1,2)^*\text{pre-}D_1 \), in general. In Example 3.10, \( X \) is \((1,2)^*\text{pre-}T_0 \) but not \((1,2)^*\text{pre-}D_1 \).
Remark 4.11. From the discussions in Sections three and four, the following implication diagram is drawn. In the diagram,

1. $(1, 2)^*\text{pre-}T_2$
2. $(1, 2)^*\text{pre-}T_1$
3. $(1, 2)^*\text{pre-}T_0$
4. $(1, 2)^*\text{pre-}D_1$

$A \rightarrow B$ (resp. $A \nrightarrow B$) represents that $A$ implies $B$ (resp. $A$ does not imply $B$).

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