

## THE PORTRAIT OF A YOUNG MAN (FROM MEMORY)

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May the reader be not surprised at the title of this paper dedicated to my teacher Myroslav Lvovych Gorbachuk on the occasion of his 70th birthday.<sup>1</sup> It happened so that my closest and most fruitful relationship with him date back 30 years. I will cover different aspects of life at that period, in particular, our collaboration, which means that although intended for a large audience, these notes occasionally require a certain mathematical background. I would also like to point out that everything written here reflects my own subjective opinion. At that time, I was at the very beginning of my career in mathematics (more exactly, at the crossroads of the paths opening ahead of me), while Myroslav Lvovych, though rather young, was making headway in his chosen area of research with confidence, inspiring people around him with his vigor and enthusiasm. Our contact has never stopped, although it became less intense over time; and other people may have more things to tell about his subsequent life and scientific achievements.

Looking back to the meetings that played a decisive role in my life, I wonder whether some of them were random. It seems to me that there were certain reasons for my closer contacts with Myroslav Lvovych. They started in my third year at the Taras Shevchenko University of Kyiv (the academic year of 1966–1967), when the students were assigned to different departments. Most of the best students chose the Algebra and the Probability Theory Departments, which were considered to be the strongest. Although I was on the list of advanced students and was free to go to wherever I wanted, my choice fell on the Mathematical Analysis Department. One of the reasons was that, like many students at that time, I dreamed of theoretical physics, but it happened so that I entered the Faculty of Mechanics and Mathematics. It seemed to me that among all mathematical subjects, functional analysis and partial differential equations were the closest to physics.

So, I was eagerly getting ready for the lectures on the theory of linear operators in Hilbert space announced for the next year. I had already learned a few things on the subject by myself, but still I wanted to have a complete picture. Then the classes began. We saw a very young man enter the lecture room, his appearance and unusual speech betraying his Western Ukrainian origin. Generally, in those days lectures in Ukrainian were rare at the University (besides the lectures in question, of all the courses I attended within its walls I also remember only lectures on algebra by V. A. Vyshensky, which were distinguished by a wonderful combination of the clarity of content and the purity of Ukrainian language). Now it may seem strange that a number of fellow students, even ethnic Ukrainians, did not relish the prospect of having classes in Ukrainian.

As for me, the fact was not in the least annoying. Although my mother tongue is Russian, I was acquainted with the Ukrainian culture, particularly the Ukrainian literature, not by hearsay. Firstly, in early childhood, the nurses of my younger brother and myself were simple country girls, with whom Ukrainian folk songs and folklore found their way to our home. Secondly, my mother, philologist by education, worked at a Ukrainian publishing house then called "Soviet Writer" and asked me frequently to help her in proof-reading. This was how I familiarized myself with Ukrainian classical literature

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<sup>1</sup>The author is grateful to A. N. Kochubei and N. S. Markova for their help in the preparation of this paper.

and a number of books by contemporary authors. When being with my mother at the publishing house, I enjoyed the sound of vivid and colorful Ukrainian speech.

Getting back to the lectures enthusiastically given by M. L., I would like to note their importance for me, because in these lectures he succeeded to show the beauty and power of the operator theory. The strongest impression was made by the proof of the spectral theorem for a self-adjoint operator. Sure, M. L.'s love for this subject cannot be explained without consideration of his "genetic memory". Indeed, his student years passed in Lviv, the city of Banach, one of the founders of the functional analysis; later Yu. M. Berezansky, who is not only an outstanding mathematician, but also a distinguished educator, became his teacher; finally, as Myroslav Lvovych told us, his mathematical evolution was greatly affected by M. G. Krein, the founder and the long-time leader of the Ukrainian school of operator theory and functional analysis. Rather good mathematical genealogy, isn't it?

Of great importance to me was the feeling that in his daily work M. L. widely used the things he told us about. It was rather tempting, and finally I asked him to set a few problems so that I could work on them on my own. Soon he proposed to me to think about the generalization of the well-known Bochner theorem on the integral representations of positive definite functions to the operator-valued case. One of the proofs of this theorem uses the famous Krein-Milman theorem on the extreme points for a convex compact set in a topological vector space. In addition, I had to try to generalize the above theorem to the case of a module over a ring of operators. Although in the end I failed to invent anything interesting, the work on this problem triggered a lot of reading and thinking in me. In particular, it was for the first time that I felt how much more complex it was to deal with modules over a noncommutative ring than with linear spaces, banal as it may seem. Have you noticed that the task was about treating purely algebraic things in the context of problems of analysis? But since then, it has been one of the attractions of the area of mathematics that I have been doing, which is to a large extent due to its "presentation" by M. L.'s as analysis with strong algebraic flavor.

When I was in my fifth year, Myroslav Lvovych proposed to me, as a possible thesis subject, a more analytical topic, namely, the expansion in generalized eigenfunctions of a Sturm-Liouville differential operator  $l[y](t)$  with unbounded operator potential of the form  $A + q(t)$  in a Hilbert space  $H$ , where  $y(t)$  is an  $H$ -valued function on the whole real axis,  $A$  is an unbounded self-adjoint operator in  $H$ ,  $q(t)$ ,  $t \in (-\infty, \infty)$ , are bounded self-adjoint operators in  $H$ . Setting this problem, M. L. recommended to begin with developing an analog of the direction functional method proposed by M. G. Krein for the spectral analysis of ordinary differential operators, which could fit for this case. The interest to this kind of problems is due, in particular, to the fact that they cover some spectral problems for hyperbolic type partial differential equations which were not so easy to study directly.

The task was facilitated by the fact that shortly before M. L. and his wife V. I. Gorbachuk had investigated analytic properties of the operator-valued solutions for the equations corresponding to this expression (see [1]). So I could use these results. It should be noted that the operators in question are associated with so-called entire operators, whose study was initiated by M. G. Krein. His lectures on the subject, given in Odessa a few years before the above-described events and noted down by his students, were processed and complemented by M. L. and his wife V. I. Gorbachuk and published as a monograph [2] years later. In general, by the end of the academic year, the needed theorem on expansion in the generalized eigenfunctions was proved, the graduation paper was written and defended, and under the suggestion of M. L., a paper on this subject was prepared and submitted for publication (this paper became my first published work).

In those days M. L. did one more thing that was important for me – he explained me how helpful regular participation in seminars of the Institute of Mathematics of the

Academy of Sciences of the former Ukrainian SSR was. I remember how happy he was to see me at the institute for the first time. Since then, for many years until my departure from Kyiv in late nineties, this institute, without exaggeration, was a second home for me (excuse me for the stock phrase, but I can't help it). All this time, I rarely passed a week without attending a few seminars and visiting the library. From late sixties through seventies I went there almost every day as if I came for a job.

M. L. was satisfied with my results, and after my graduation from the university he said that he would willingly become my supervisor if I entered the PhD program, all the more so that I graduated with distinction. But that was not to be. It should not be forgotten that we lived in the country where antisemitism was the state policy. I had already faced it when I had tried to enter Moscow State University. I had not been admitted there in spite of the high score at the entrance exams. After my graduation from the Taras Shevchenko University of Kyiv, the situation was the same. More than once M. L. visited the then dean of the Faculty of Mechanics and Mathematics I. I. Lyashko, asking him to sign the recommendation for me to enter a PhD program (since I had a diploma with distinction, I was entitled to it). At last, failing to obtain the recommendation, he offered to continue teaching me outside the formal PhD program, with a view to a prospective PhD thesis. It is needless to say how vital this support was for my career. This case was not unique for M. L. Three years later, another M. L.'s student, A. N. Kochubei, found himself in the same situation. He was compelled to earn his living by programming (later, in applied physics) doing mathematics as a hobby for years. And only many years later, when the times changed, through M. L.'s and Yu. M. Berezansky's initiative, he was admitted to the Institute of Mathematics. Still later he became the head of a department. M. L. also supported talented mathematician A. Reznikov in a very hard period of his life.

Where did his compassion and desire to help come from? Maybe that can be understood if you look back into M. L.'s childhood in Sub-Carpathian countryside, where people of various ethnic origin and confessions had lived side by side, and where the tolerance and the desire to understand and support one each other had been natural. That was for a good reason that he could not stand idly by watching a dull totalitarian state machine of the former USSR maim and crush peoples lives. And the reality added regularly new and new cases of such kind. Here are two of them that I have witnessed.

In 1968 a number of artists, writers, scientists etc. signed the letter in defense of the people who had been arrested shortly before, the so-called "Ukrainian bourgeois nationalists", in fact, those who fought against uninterrupted ruthless suppression of Ukrainian culture. Among the mathematicians who had signed this letter were V. A. Vyshensky, A. V. Skorokhod, Yu. D. Sokolov and Yu. M. Berezansky. Doing this took great courage. The standard response of the ruling system was public punishment of the "signers". Since three of them (except Yu. D. Sokolov) were professors at the University, a faculty meeting was organized, with the preordained decision to condemn them. Yu. M. Berezansky and A. V. Skorokhod, who taught at the University part-time, were banned from teaching. As a result, it was the students who suffered most, since they were deprived of the possibility to hear lectures of the leading Ukrainian mathematicians. My mother told me that in her publishing house a number of writers had been punished, too. Just at that time I was working on my graduation paper and frequently contacted M. L. I remember him complaining that because of these events, in particular, the persecution of his teacher, he could not focus on mathematics. Of course, at that time he did not take the liberty to discuss openly such things, but I felt well his anxiety about the proponents of the Ukrainian culture and science who were being through dreadful ordeal those days.

A few years later there was another event, also typical for those days. The thesis for Doctor of Science degree (which is the equivalent of habilitation) by D. Z. Arov, one of the best M. G. Krein's disciples, was sent for review to one of the departments of the Institute of Mathematics. The report was unfavorable. Strictly speaking, it was no wonder, given the department's reputation; it was just to this end that the dissertation was sent to the department whose profile did not match the dissertation subject. I would like to avoid naming the people some of whom are already dead, including the head of the department. In an effort to put on a semblance of objectivity, the department organized an open meeting to approve the report and invited the thesis author D. Z. Arov, as well as M. G. Krein who came to Kyiv to support his disciple. When I saw the announcement, I decided to attend this meeting, since I expected to submit my own thesis a few years later, though it was too clear to me that I would hear nothing good. Indeed, when the reviewer took the floor, his report presented a long list of "mistakes" that were ridiculous. But both Arov's well-founded arguments and Krein's in-depth comments remained unheeded. Why care for the opinion of the prominent scientist to whom P. Lax and R. Phillips dedicated their book with the following inscription: "This book is dedicated to M. G. Krein, one of the mathematical giants of our century, as a tribute to his unusually wide and deep contribution into mathematics. As all the analysts, we are much obliged to him". Why care for this while they were obliged to carry out the instructions "from above"? Of course, the negative report was approved, and Arov's thesis was rejected.

Although I failed to enter a PhD program, M. L. supported me and agreed to work with me as an informal supervisor. My father helped me to get a job at the Aviation Engineering Military School where I had to deal with problems of mathematical modeling of technical systems. Fortunately, a little time and energy remained for mathematical studies that I was keen to pursue. At the same military school, G. I. Kac, whom I also regard as my teacher, held a professor position at the Mathematical Department. In the collaboration with him, the theme of my present work was being gradually "crystallized", which not in the least hindered my informal studies under the supervision of M. L., who proposed to me to consider a large class of spectral problems for expression  $l[y](t)$ , where the operator  $A$  could be either positively or negatively definite, that is, the differential expressions of both elliptic and hyperbolic types were acceptable.

By that time, the paper by M. L. and V. I. Gorbachuks had been published, where the self-adjoint boundary value problems were considered for an elliptic type equation on a finite interval. As it was shown there, like in the case of elliptic type partial differential equations, the solutions of such equations have an increased smoothness inside the interval but their smoothness at the ends of the interval is lowered (see [1]). I noticed that in considering the similar problems on the whole real axis, the above results allowed to apply the so-called direct methods of spectral analysis described in the book by I. M. Glazman [3], which could serve as basis to obtain a number of the self-adjointness and the spectrum discreteness tests for the related operators (see [4,5]). The other advance consisted in the description of self-adjoint boundary value problems in the hyperbolic case on a finite interval, obtained by analogy with the mentioned work of M. L. and V. I. Gorbachuks, but on the basis of "trickier" formulas for generalized boundary values (see [6]).

Though pleased with our achievements, I was aware of the fact they were no more than isolated results. I wanted to obtain something more significant for my thesis. Little by little, we came up with the idea that would do was only systematic investigation of spectral problems for expressions of the same form but with variable positive or negative definite coefficient  $A = A(t)$ . I managed to find in the literature the methods for construction of the solutions to such equations. In particular well-known Kyiv mathematician Yu. L. Daletsky was dealing with similar problems in hyperbolic case. Step-by-step I

succeeded to carry out the whole planned program starting from the study of smoothness properties of generalized solutions of these equations and the description of the certain classes of spectral problems on a finite interval, and finishing by spectral analysis of the corresponding operators on the semi-axis and the whole real axis.

All this work was going on in very favorable and creative atmosphere. There was a special local seminar organized by M. L. for his graduate students and colleagues, where all the participants exchanged their ideas freely. For instance, the abstract concept of a boundary value space developed by that time by A. N. Kochubei for symmetric operators, and the resulting general approach to the description of various boundary value problems, turned out to be very useful for many of us. Then the most important results were reported at Yu. M. Berezansky's seminar of Mathematical Analysis Department of the Institute of Mathematics, which was in fact the city seminar on functional analysis and partial differential equations. This was how the foundations were being laid for the scientific school created by M. L. soon after these events. Within it he fostered a number of students and disciples and contributed to the emergence and development of many fruitful ideas.

In 1974 it was decided that I could try to submit my candidate thesis (the equivalent of PhD). Though the memories were still raw about numerous dissertations rejected only because of a "poorly sounding" (that is Jewish) family name or patronymic, it looked like that campaign was on the wane. The thesis was submitted, the defense went smoothly, to a great extent owing to the man who was teaching and supporting me over these years both by words and actions.

After the defense of the thesis, during some years I continued tackling problems related to the thesis subject. I considered operator differential equations with degeneration and mixed-type ones and investigated solvable, not necessarily self-adjoint boundary value problems. But in the course of time, other mathematical interests began to gain the upper hand. It is known that with every first-order differential expression, a semigroup of operators can be associated. The experience in the area proposed to me by M. L. showed that in the similar way the second-order differential expressions generate more complex families of operators, so-called generalized translation operators. Earlier J. Del-sarte, B. M. Levitan, and from the very beginning of the fifties Yu. M. Berezansky and S. G. Krein (in Kyiv) took interest in these objects (they called them the hypercomplex systems with continuous basis). Later this subject began to evolve in France and the USA as the theory of hypergroups. On the other hand, G. I. Kac taught me to work with noncommutative algebraic structures, and approximately since late seventies I began to engage systematically in the harmonic analysis and the theory of representations of general enough hypergroups. Yu. M. Berezansky supported me very much in this activity. Of course, in so doing, I moved away from the field of operator differential equations which is being developed up to now by M. L. Gorbachuk, his collaborators and PhD students. Nevertheless, to be fair, I would like to note that some important examples of hypergroups, in particular, unbounded and not necessary satisfying the positive definiteness condition, were constructed with the essential use of the techniques acquired in the years of my work at the candidate dissertation.

From our relationship outside mathematics, I remember M. L. singing, especially Ukrainian folk songs. My wife and I were greatly impressed with his rendering of the "Streltsi songs" (the songs of Ukrainian regiments of World War I), which were always performed by him with great pleasure. He has a strong voice, whose powerful sound overrides easily the other guests singing at a party table. Being well aware of his gift, he does not hesitate to solo being confident that the guests will listen to him with pleasure. Here is one example on the music theme. In summer 1991, my Paris colleague Michel Enock visited Kyiv. On this occasion, all my family, friends and collaborators came

around. M. L. and his wife were also invited. After meal we began to sing, as was our habit, which turned out to be a novelty for my French colleague. After the guests had left, he was still under the impression from what he saw and heard, especially from M. L.'s singing. A little later, he remarked pensively that, unfortunately, he could not even imagine for his teacher J. Dixmier to visit him at home without ceremony, and sing with the guests at the table, moreover, perform a solo.

It happened so that since the mid-nineties I come to Kyiv not so frequently and look at the events in my homeland mainly from afar. Over the years of independence, the feeling of lack of freedom, that used to spoil our life in the former USSR, disappeared. But the progress in independent economic, cultural and scientific development of Ukraine is slow and hard. It is not astonishing, because for more than three centuries Ukraine (or at least its larger part) lay beneath the yoke of two vast empires, first Russian then Soviet, that hunted down the smallest signs of independence and crushed them ruthlessly. Fortunately, throughout the history of this country there always were a lot of people who managed to maintain, against all odds, their insuperable desire of freedom and independence. There are many such people just now, and M. L. is one of them. Through their struggle the historical truth will prevail over all current problems. This will be not easy and simple, but this will certainly happen. One cannot say better than Lina Kostenko has:

In the historical cellars  
The wines of truths will settle  
And truth's burning honeys  
With blood will painfully rise.

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Received 28/01/2008