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IN MEMORIAM ISRAEL GOHBERG

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On October 12, 2009, at the age of 81, Israel Gohberg passed away in Meir Hospital in Kfar Saba, not far from his hometown Ra'anana in Israel. At that time, already for many years, he was a dialysis patient and had several bypasses. Nevertheless, he continued to work on a high level, and for many of his colleagues and friends his death came unexpected.

Israel Gohberg will be remembered as a wonderful and charismatic mathematician. His style and fine mathematical taste inspired his fellow mathematicians and attracted coworkers. His mathematical papers and books are beautifully written and usually carry many illustrative examples and unexpected applications. They are widely known and often quoted, also outside mathematical circles, in particular by physicists as well as engineers from electrical engineering and control and system theory.

Mathematical legacy. Gohberg's mathematical work is fundamental and extensive. More than 450 mathematical articles carry his name. He supervised 40 Ph.D. students and coauthored 27 books of which the last appeared in 2010. The first books were in Russian and have been translated into English, French and German. Except for two attractive little books on elementary geometry and combinatorial problems, which he wrote together with Vladimir Boltyansky (1965, 1971), his contributions belong to the fields of analysis, operator theory, linear algebra, numerical analysis, and control theory. His work is leading in the following research areas: Wiener-Hopf and singular integral equations and their discrete analogues, Toeplitz equations; the theory of nonselfadjoint operators; spectral theory and factorization of matrix and operator functions; and inversion problems for structured matrices. His mathematical influence was profound and far reaching.

In addition to his research activities, Israel Gohberg made many contributions to the mathematical community. He was the founder in 1978 and the editor-in-chief of the journal *Integral Equations and Operator Theory* and of the book series "Operator Theory: Advances and Applications." The journal has more than 50 volumes and the book series over 200 titles of which several are directly related to mathematical topics that are of interest to engineers. In Tel-Aviv he organized the biennial Toeplitz Conferences. Together with J. W. Helton he initiated in 1981 a series of international workshops on operator theory and its applications (IWOTA). At the beginning these workshops were biannual and meant as satellite conferences to the engineering conferences on Mathematical Theory of Networks and Systems. They are now held annually, and the series is truly international: IWOTA workshops have been held in America, Europe, the Middle East, Africa, and Asia. All these activities – the journal, the book series, the workshops – Israel Gohberg managed up to the end of his life in spite of increasing health problems. He was a true leader of a mathematical school.

Childhood, introduction to mathematics. Israel Gohberg was born in 1928 in Tarutino, in Bessarabia, which at that time was part of Romania; it became part of the Soviet Union in 1940. The latter transition was a disaster for the Gohberg family. His father was arrested on the grounds of being a "capitalist exploiter" and disappeared in

the Gulag from where he did not return. In the beginning of the Second World War his mother Clara and her two children Israel (Izea) and Fanja had to flee from the advancing German army. They spent the war time in a village close to Frunze (Bishkek), the capital of Kyrgyzstan. At the local high school he had an excellent math teacher, a recent graduate from the University of Warschau, a student of K. Borsuk, who aroused his interest in mathematics.

After high school Israel Gohberg studied at the Pedagogical Institute in Frunze, where his interest in mathematics was developed further. In 1947 he received a Stalin fellowship, a great honor and financially very welcome. However, the program of the Pedagogical Institute focussed on the education of further teachers and not on mathematical research. Therefore, in 1947, Gohberg gave up the Stalin fellowship and left Frunze to study mathematics at the University of Kishinev (Moldova). At this university a young teacher, a former student of S. G. Mikhlin, told him about singular integral equations, Fredholm theory and Fredholm index. Later Gohberg would contribute to these topics extensively.

Making a career as a mathematician. From 1951–1953, during the years of Stalin's plot against the medical doctors, Gohberg worked at an institute for the education of high school teachers in Soroki, a small town in the north-eastern part of Moldova. When the director of the institute in Soroki, a mathematician, was promoted to director of the Pedagogical Institute in Beltsy, which had a university status, he invited Gohberg to join him in this transition. Gohberg worked in Beltsy from 1953–959, the last years as chairman of the mathematics section. Gohberg obtained his first doctoral degree in 1954 in Leningrad. His salary doubled and he could invite his mother and sister to come over from Kyrgyzstan.

In Leningrad Gohberg also met his future wife Bella Elkina, a friend of his sister; they married in 1956 and got two daughters. A happy marriage and a happy family which he enjoyed his whole life. In the last 10 years or so when his health problems were increasing, Bella, a medical doctor, always joined him on his foreign journeys. Without her it would have been very difficult, if not impossible, to make these trips.

In 1959 Gohberg returned to Kishinev as professor and head of the functional analysis department in the Mathematical Institute of the newly organized Moldavian Academy of Science. In 1964 he obtained the second doctoral degree in Moscow. From 1963–1973 he held a part-time professorship at the University of Kishinev. In 1970 he was elected a corresponding member of the Moldavian Academy, at that time an unusual honor for a mathematician of Jewish origin.

The cooperation with M. G. Krein. In the period 1950–1970 Israel Gohberg often stayed for two or three months in Odessa for joint work with the great master of analysis, Mark G. Krein. The collaboration with Krein has been very important to Israel Gohberg. Although not officially a student of Krein, he would often speak about Krein as his teacher and about what Krein had taught him. Arguments were sometimes settled with a phrase like: "I learned from Krein ..."; further discussion was then out of the question. In his "Mathematical Tales" [6] Gohberg wrote the following story about his first meeting with Krein:

I didn't even know the telephone number of Krein, but I knew where he was working; I already knew some of his work. He was at the Marine Engineering Institute in Odessa. This was a semi-military academic institution, so the professors wore uniforms and held ranks, which were denoted by the braid on their uniforms. An ordinary person could not enter; one needed to have some special identity card, which I for sure did not have, but I saw some students running in waving something in their hands. I also ran in waving something, and so I came to Krein's office. At that time he held the chair in theoretical mechanics. Some time earlier he had been dismissed from the Odessa University. I came

up to the secretary sitting at a deck and said that I would like to meet Professor Krein. She said, "He will be here in an hour. There will be a seminar." I waited. After an hour I saw two men with all this braid and insignia on their uniforms – colonels, captains, whatever – two men, one looked very ordinary and the second fat and distinguished, who I was sure must be Krein. I went up to him and asked, "Are you Mark Grigorievich Krein?" "No", he said, "This one is." Krein invited me to his seminar and I listened to his talk. It was a very interesting talk on the behaviour of a stretched string, on extreme mass distributions. After his talk, he spoke to me. I showed him my first results; they were about Fredholm operators and index. He invited me to his home in the evening, and there we had our first long conversation. He drew my attention to two topics. The first was commutative normed rings, today we would say commutative Banach algebras. I did not know anything about it. "You dont know about this" he said. This was in 1950. He said this was essential for a mathematical education. I felt terrible, non-educated. Then he showed me a couple of papers which had just appeared on infinite Toeplitz matrices. He said that they looked interesting to him.

The collaboration with Krein led to many outstanding contributions in integral equations, including two fundamental papers in *Uspekhi Mat. Nauk*, the most prestigious mathematical journal in the former USSR, on Fredholm theory (1957) and on systems of Wiener-Hopf equations (1958). Their joint work on the theory of nonselfadjoint operators culminated in two books: *Introduction to the theory of linear nonselfadjoint operators* (1965) and *Theory and applications of Volterra operators* (1967). Additional contributions were on factorization theory for operators, which is now used in many different areas, such as system theory, probability theory, integral equations, and the theory of singular values for bounded operators, which turned out to be of use in H^∞ interpolation theory through the connection with infinite Hankel matrices.

The years in Kishinev. Gohberg's work on singular integral equations was innovating, in particular, the use of the at that time relatively new Gelfand theory of Banach algebras. This work was the basis for his first dissertation, and he continued this work in Kishinev with his student Nahum Krupnik. They found the correct description of the symbol of singular integral equations of which the defining functions had jump discontinuities. Surprisingly, the symbol had to be a matrix function, even when the coefficients were scalar. The famous formulas of Gohberg-Semencul and Gohberg-Heinig for the inverse of a finite Toeplitz matrix and the continual analogues of these formulas all date from the Kishinev period. These inversion formulas received a great international status. They are important in numerical linear algebra. Nowadays they are viewed as predecessors of the modern theory of structured matrices and operators. All the Gohberg-Krupnik and Gohberg-Heinig papers dating from the Kishinev period are written in Russian but just this year they became available in English [10]. Construction of inverses also played an important role in his joint work with Fel'dman on the projection method.

In Kishinev, Israel Gohberg attracted many strong doctoral candidates who officially or informally became his students and later coworkers. Two came from the former DDR: Georg Heinig, who passed away in 2005, was one of them, and the other was Jürgen Leiterer, presently professor at the Humboldt University in Berlin. When the thesis of Leiterer was finished they started to work on factorization of matrix and operator functions. They realized that methods from several variable complex function theory are very useful in studying factorization of holomorphic operator functions. Their first joint papers appeared in the beginning of the seventies and 20 years later, when the Iron Curtain had disappeared, they returned to this project which resulted in a book that was finished just before Gohberg passed away.

In April 1964 Seymour Goldberg, from the University of Maryland at College Park in the USA, visited Gohberg in Kishinev. A very unusual event: a visitor from the West.

It resulted in a life long friendship. They coauthored several books, including the very successful textbook *Basic Operator Theory*. Goldberg passed away in 2004. In 1989 in his "First Meeting" [8] he wrote:

What amazes me is that in spite of the hardships encountered by Israel, he remains a warm, kind and gregarious human being. He is an inspiration and a delight to all those who know him.

Refusenik. In the end of the 60ties, beginning of the 70ties there was an increase in anti-semitism in the Soviet Union, also in mathematical circles. Although anti-semitism did not exist officially, it became more and more difficult for Jewish students to enter a Ph.D. program and for Jewish mathematicians to publish their work. The second doctorates of excellent Jewish mathematicians were not approved. In the period 1968–1984 no Jewish mathematician was elected as a full member of the USSR Academy of Sciences in Moscow. Gohberg started to plan emigration to Israel. In his "Mathematical Tales" [6] he wrote:

With the agreement of all my family I started to plan our emigration. At the same time I also took the necessary steps at my place of work. I refrained from taking on any new students, and I saw to it that almost all of my students would finish their theses before I applied for emigration. By the way, two of my Ph.D. students from Kishinev University defended their theses after I applied for an exit visa, my name as instructor was replaced by another, and the students were not allowed to mention me at all. I stopped working (part time) at Kishinev University. I tried to complete joint work which was in process, and many other things had to be taken care of. I followed closely the progress of the emigration process of people in a position similar to mine.

The application of the Gohberg family for a visa to Israel was rejected. Gohberg became a refusenik. About this period he wrote in his "Mathematical Tales" [6] the following:

During this difficult period of being a refusenik my family and I received a lot of support from the West. A businessman from England, Gerald Wise, often called us an the telephone. This gave us great moral support. We felt as if the entire Jewish people, the entire free world, was behind him. Gerald informed my colleagues in the West of my difficulties, and shortly after I received an important call from Chandler Davis, who became very active in my case. We received letters and telephone calls from other colleagues and friends in the United States and Israel. I was told that I was also supported at a meeting of the Bourbaki seminar.

During the darkest period, when the future of my family and myself was in real danger, two couples from the United States visited Kishinev for a few days in order to help us and other refuseniks. One couple was the New York lawyer, Alvin Hellerstein and his wife, and a special prosecutor of the State of New York, Maurice Nadjari and his wife. It is impossible to describe how much this visit meant to us. My family and myself will always remember with gratitude all those who helped us in our struggle to emigrate from the Soviet Union to Israel.

After emigration. In 1974 a new life started; the world was open to Gohberg. He became a professor at Tel-Aviv University, where he held the Nathan and Lily Silver Chair in Analysis and Operator Theory until his retirement in 1998. In the years after his emigration, he held part-time professorships at various other institutions in Europe, North America, and Israel, including the VU University in Amsterdam (from 1976–2006), the Weizmann Institute (1975–1983), and the University of Maryland at College Park (1980–2002). Moreover, for various extended periods of time he held semi-permanent positions at the University of New York at Stony Brook, the University of Calgary in

Alberta, Canada, and the University of Georgia at Athens, Georgia, and later also with a prestigious Humbold Fellowship at various places in Germany.

After his departure from the Soviet Union, Israel Gohberg started several major new research projects, first at Tel-Aviv, where he led a group in analysis and operator theory, in Amsterdam in collaboration with M. A. Kaashoek, in Calgary in collaboration with P. Lancaster and L. Rodman, and at College Park in collaboration with S. Goldberg, R. Ellis, and D. C. Lay. Typical for this period and truly remarkable is his interaction with control and electrical engineers. Inspired by these connections he developed new directions in operator theory, which resulted in new constructive operator theory methods with substantial mathematical contributions to control theory. Four main topics are involved.

(i) Spectral analysis of matrix and operator polynomials and, more generally, of holomorphic matrix and operator functions, with a new view on linearization and with applications to differential and difference equations, resultant and bezoutian and orthogonal polynomials.
(ii) The state space method to treat problems in analysis and operator theory, with significant contributions to the theory of Wiener-Hopf factorization and modern control problems, such as sensitivity minimization, model reduction, and robust stabilization.
(iii) Completion and extension problems for partially given matrices or operators, with applications to eigenvalue completion problems, to stabilization problems in mathematical system theory.
(iv) Contributions to numerical analysis and numerical linear algebra, including parallel algorithms for semiseparable integral operators, which are viewed as input-output operators of systems, and finite section and projection methods for convolution operators, fast algorithms for structured matrices and for their block versions.

In the context of the present article it is impossible to describe the details of his work. I will single out only one topic "The state space method" which was one of the main themes of his work in Amsterdam.

The state space method. In operator theory it is very common to analyze concrete classes of integral and differential equations in terms of analytic functions which are in a natural way related to the equations involved. Analyzing Toeplitz equations in terms of the associated symbol is an example where such a reduction is very useful. In 1976 when Gohberg was in Amsterdam, he suggested as a next step a second reduction, which would become one of the highlights in the work he, Harm Bart and I were going to do jointly. His suggestion was on the one hand inspired by the theory of characteristic functions for operators close to selfadjoint or unitary operators, and on the other hand by the spectral analysis of matrix and operator polynomials, work he had been doing just after his emigration to Israel with Peter Lancaster and Leiba Rodman.

The new step Gohberg had in mind is based on the observation that the analytic functions appearing in operator theory often can be analyzed in terms of three or four operators which are often much simpler than the original operator. It uses the idea of state space realization which is common in mathematical system and control theory, and it views the analytic functions involved as transfer functions of linear input-output systems.

To illustrate the idea let us consider a Wiener-Hopf integral operator:

$$f(t) - \int_0^\infty k(t-s)f(s)\,ds = g(t), \quad 0 \le t < \infty. \tag{1}$$

Assume that its symbol $W(\lambda) = I_m - \int_{-\infty}^{\infty} e^{i\lambda s} k(s)\, ds$ is a rational $m \times m$ matrix function. Then the symbol has no pole at infinity and a classical realization theorem due to Kalman tell us that the symbol can be written as

$$W(\lambda) = I_m + C(\lambda I_n - A)^{-1} B. \tag{2}$$

Here I_m and I_n are identity matrices of order m and n, respectively, and A, B and C are matrices of appropriate sizes. In this way the analysis of the integral equation (1) is reduced to a linear algebra problem, which is solved by the following theorem.

Theorem 1. *Equation* (1) *is uniquely solvable in the Lebesgue space of all integrable* $\mathbb{C}^n$*-functions on* $[0, \infty)$ *if and only if the following two conditions are fulfilled:*

- *the matrix* $A - BC$ *has no real eigenvalue;*
- *the subspace* M *spanned by all the eigenvectors and generalized eigenvectors of* A *corresponding to the eigenvalues in the open upper half plane and the subspace* $M^\times$ *spanned by all the eigenvectors and generalized eigenvectors of* $A - BC$ *corresponding to the eigenvalues in the open lower half plane form a direct sum decomposition of* $\mathbb{C}^n$*, that is,* $\mathbb{C}^n = M \dot{+} M^\times$.

In that case the unique solution of (1) *is given by*

$$f(t) = g(t) + \int_0^\infty \gamma(t,s) g(s)\, ds, \quad 0 \le t < \infty,$$

where the resolvent kernel function $\gamma(t,s)$ *is obtained in the following way:*

$$\gamma(t,s) = \begin{cases} iCe^{-it(A-BC)} \Pi e^{is(A-BC)} B, & 0 \le s < t < \infty, \\ -iCe^{-it(A-BC)} (I_n - \Pi) e^{is(A-BC)} B, & 0 \le t < s < \infty. \end{cases}$$

Here Π *is the projection of* $\mathbb{C}^n$ *along the space* M *onto the space* $M^\times$.

The above is just one example. The state space method has been used to solve problems in analysis, ranging from infinite systems of linear Toeplitz equations, interpolation theory and transport problems in astrophysics to canonical differential systems and nonlinear integrable partial differential equations; see the review articles [1, 3, 7].

Honors. Israel Gohberg's achievements and activities did not go unnoticed. Among the many honors that have marked his career, we mention his election as a Foreign Member of the Netherlands Academy of Arts and Science in 1985, in the seat of the late Marc Kac, the Landau Prize (1976), the Rothschild Prize in Mathematics in 1986, the prestigious Humboldt Research Prize in 1992, the Hans Schneider Prize in Linear Algebra in 1994, the M. G. Krein prize of the Ukrainian National Academy of Sciences in 2007, and more recently his election as SIAM Fellow in 2009. He received honorary doctorates from universities in Germany (Technical University Darmstadt, 1997), Austria (Technical University Vienna, 2001), Romania (Timisoara, 2002), Moldova (Kishinev and Balti, both 2002), and Israel (Technion Haifa, 2008).

Gohberg and Friends. Israel Gohberg's exceptional life and his many mathematical achievements are described in greater detail in the book *Israel Gohberg and Friends* [4] which appeared on the occasion of his 80th birthday in 2008. This book also contains mathematical tales written by Israel Gohberg, about his mathematical life and his vision on mathematics, sixteen articles written or coauthored by him about his relation with some of his colleagues and friends, as well as an article written by his two daughters narrating how their father introduced them to mathematics. Gohberg was a great mathematician and an exceptional friend. He will be dearly missed by many of us.

Acknowledgments. Further information, from different points of view, can be found in the obituaries [2, 5, 9]. I have freely used information from these texts.

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