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ON GENERALIZATION OF THE FREUDENTHAL'S THEOREM FOR COMPACT IRREDUCIBLE STANDARD POLYHEDRAL REPRESENTATION FOR SUPERPARACOMPACT COMPLETE METRIZABLE SPACES

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Abstract. In this paper for superparacompact complete metrizable spaces, the Freudenthal's theorem for compact irreducible standard polyhedral representation is generalized. Furthermore, for superparacompact metric spaces the following is strengthened: 1) the Morita's theorem about universality of the product $Q^\infty \times B(\tau)$ of Hilbert cube Q^∞ to generalized Baire space $B(\tau)$ of the weight τ in the space of all strongly metrizable spaces of weight $\leq \tau$; 2) Nagata's theorem about universality of the product $\Phi^n \times B(\tau)$ of the universal n-dimensional compact Φ^n to $B(\tau)$ in the space of all strongly metrizable spaces $\leq \tau$ and dimension $\dim X \leq n$.

In what follows, by a space we mean a topological space, a compact is a metrizable bicompacts a mapping is used to mean a continuous mapping of spaces. Furthermore, a polyhedron we mean a spatial (generally speaking, infinite) simplicial complex (see [1], Chapter 3, § 2) in metrizable topology.

We give main definitions and some necessary concepts for this paper.

Definition 1. [1]. a) A system ω of subsets of the set X is called star countable (finite), if every element of the system ω is intersected at most at a countable (finite) number of elements of this system; b) a finite sequence of subsets $M_0, \dots, M_s$ of the set X is called a chain connecting the sets M_0 and M_s, if $M_{i-1} \cap M_i \neq \emptyset$ for all values $i = 1, \dots, s$; c) a system ω of subsets of the set X is called an enchained set, if for all sets M and M' of this system there exists a chain of elements of the system ω such that the first element of the chain is the set M, and the last is the set M'; maximal enchained subsystems of the system ω are called components of overlapping (or components) of the system ω.

It is known that [see [1], Chapter 1, § 6] components of the star countable system ω are countable.

Definition 2. [2]. (a) A star-finite open covering of a space is called finite-component if all components of the overlapping are finite;

(b) A space is called superparacompact if, given an open covering of the space, a finite-component covering can be inscribed in this open covering;

(c) Hausdorff superparacompact spaces are called superparacompacta.

Definition 3. [1]. (a) A finite covering $\omega = \{O_1, \dots, O_s\}$ of a space X is said to be *irreducible*, if no proper subcomplex N' of the nerve (see [1], Chapter 4, § 2) N_ω of the covering ω is not a nerve of a covering smaller than ω (i.e. of a covering inscribed into covering ω);

(b) a complete finite complex K (see [1], Chapter 3, § 2), elements of which are disjoint open simplexes of a given space R^n is called *triangulation*, lying in R^n.

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(c) the mapping f of the space X in body $\widetilde{N}_\omega$ of triangulation N_ω is *irreducible* with respect to this triangulation, if it is essential (see [1], Chapter 3, § 5) on the preimage of every closed simplex of this triangulation;

(d) Let the triangulation $N = N_\omega$ be a geometrical realization into R^m of the nerve of the covering $\omega = \{O_1, \ldots, O_s\}$ of the space X and e_i be a vertex of the nerve N corresponding to an element O_i of the covering ω.

The mapping f of the space X to polyhedron $\widetilde{N}$ (see [1], Chapter 4, § 1]) is called *canonical* with respect to covering ω, if the preimage $f^{-1}Oe_i$ of every star Oe_i contains in O_i;

(e) the space with a σ-star finite base is called (see [1], Chapter 6, § 3) *strongly metrizable.*

Definition 4. A finite component covering ω of the space X is called *irreducible*, if all its components coupling are irreducible (i.e. the components ω_λ of the covering ω are irreducible covering of own bodies $\widetilde{\omega}_\lambda$).

Definition 5. [1]. *By a product* of two systems of sets $\alpha = \{A\}$ and $\beta = \{B\}$, we call the system of sets $\gamma = \alpha \wedge \beta$, by elements of which are all (denoted) of sets of the form $A \cap B$, where $A \in \alpha$, $B \in \beta$.

Proposition 1. *Into any open covering of a superparacompact Hausdorff space X, an irreducible finite-component open covering can be inscribed.*

Proof. Let ω be any open covering of a superparacompact Hausdorff space X. Since the space X superparacompact, without loss of generality, the covering ω can be assumed to be finite component. Since every component ω_λ, $\lambda \in L$, of the covering ω is finite, and their body $\widetilde{\omega}_\lambda$, $\lambda \in L$, is open-closed in X, in the covering ω_λ of the set $\widetilde{\omega}_\lambda$, an irreducible open covering ω^*_λ can be inscribed (see [1], Proposition 2, Chapter 4, § 2). Then the system $\omega^* = \cup\{\omega^*_\lambda : \lambda \in L\}$ is an irreducible finite component open covering of the space X, inscribed into ω. □

Remark 1. a) If $\omega = \{O_\alpha, \alpha \in A, |A| = \tau\}$ is a finite component open covering of the space X, then the body $\widetilde{gN_\omega}$ of the standard geometric realization gN_ω in the Hilbert space R^τ of the nerve N_ω of the covering ω (standardization of the realization means that the vertices of the triangulation gN_ω are located in unit points of the space R^τ) is a discrete subcompact polyhedron $\widetilde{gN_{\omega_\lambda}}$, being the body of a realization of the nerves N_{ω_λ} of the component ω_λ of the covering ω.

We note that the polyhedron $\widetilde{gN_\omega}$ is superparacompact (see [3], Proposition 2). If, in addition, the covering ω has multiplicity $\leq n+1$, then the polyhedrons $\widetilde{gN_{\omega_\lambda}}$ is not more than n-dimensional and thus $\dim \widetilde{gN_\omega} \leq n$.

b) From the theorem about canonical mappings (see [1], Chapter 4, § 1, Theorem 1) when transferring to bodies of components of a finite component covering, it is easy to get following.

Proposition 2. *Let $\omega = \{O_\alpha, \alpha \in A\}$ be any finite component open covering of a normal space X with nerve N_ω, realized in the triangulation form; it is possible to find a subcomplex N'_ω of the nerve N_ω and a mapping $f : X \to \widetilde{N}_\omega$, which is canonical with respect to ω, such that the image fX is a polyhedron $\widetilde{N}'_\omega \subseteq \widetilde{N}_\omega$ and every principal simplex of the complex N'_ω is covered essentially.*

Proposition 3. *For any finite component irreducible covering $\omega = \{O_\alpha, \alpha \in A, |A| = \tau\}$ of a normal space X, arbitrary canonical mapping of the space X into the body $\widetilde{gN_\omega}$ of the standard geometrical realization gN_ω in the Hilbert space R^τ of the nerve N_ω of the covering ω is irreducible with respect to the triangulation gN_ω.*

Proof. Let f be any canonical mapping (with respect to the covering ω) of the space X into the body $\widetilde{gN}_\omega$ of the standard geometrical realization gN_ω in the Hilbert space R^τ of the nerve N_ω of the covering$\omega = \{O_\alpha, \alpha \in A, |A| = \tau\}$. Then, according to Remark 1, the body $\widetilde{gN}_\omega$ of the standard geometrical realization gN_ω into R^τ of the nerve N_ω of the covering ω is a discrete sum of compact polyhedrons $\widetilde{gN}_{\omega_\lambda}$, being bodies of a realization of the nerves N_{ω_λ} of the components ω_λ of the covering ω. Suppose $f_\lambda = f : \widetilde{\omega}_\lambda \to \widetilde{gN}_{\omega_\lambda}$ for any $\lambda \in L$. It is clear that every mapping f_λ, $\lambda \in L$, is canonical (with respect to the covering ω_λ).

Since the covering ω is irreducible, all its components ω_λ, $\lambda \in L$, are irreducible, according to Definition 4, and therefore each canonical mapping $f_\lambda : \widetilde{\omega}_\lambda \to \widetilde{gN}_{\omega_\lambda}$ is (see [1], Chapter 4, § 1, Proposition 3) irreducible. Then the canonical mapping $f : X \to \widetilde{gN}_\omega$, as a combination [4] of irreducible mappings $\{f_\lambda, \lambda \in L\}$, is irreducible with respect to the triangulation gN_ω. □

Later in this work by a triangulation in the Hilbert space R^τ we mean either the standard geometrical realization gN_ω of the nerve N_ω of a finite component covering ω of a normal space, or its subdivision (see [1], Chapter 3, § 2, Section 5) $(gN_\omega)^*$, which for each component ω_λ of the covering ω coincides with some (multiple, and also the multiplicity depends on the component ω_λ) barycentric subdivision (see [1], Chapter 3, § 2, Section 6) of the triangulation gN_{ω_λ}.

Remark 2. a) Let $\omega = \{O_\alpha, \alpha \in A, |A| \le \tau\}$ be a finite component $(n+1)$-multiplicity covering of a normal space X, $\widetilde{gN}_{\omega_\lambda}$ is the body of the standard geometrical realization gN_ω in the Hilbert space R^τ of the nerve N_ω of the covering ω and $\varepsilon > 0$. We take such natural s that $\left(\frac{n}{n+1}\right)^s \sqrt{2} < \varepsilon$. Then in virtue of all k-dimensional simplexes of the triangulation gN_ω are isometric and the relation $\left(\frac{k}{k+1}\right)^s \sqrt{2} \le \left(\frac{n}{n+1}\right)^s \sqrt{2}$, $k = 1, 2, \ldots, n$, it follows that all simplexes of the subdivision $(gN_\omega)^*$, being s-multiplicity barycentric subdivisions of the triangulation gN_ω, have diameter $< \varepsilon$.

b) Let ω_1 be a finite component open covering of the normal space X, gN_{ω_1} a standard geometrical realization of the nerve N_{ω_1} of the covering ω_1 in the Hilbert space R^τ and f_1 be a canonical, with respect to the covering ω_1, mapping of the space X into $\widetilde{gN}_{\omega_1}$. Let $(gN_{\omega_1})^*$ be a triangulation of the polyhedron $\widetilde{gN}_{\omega_1}$, being a subdivision of the triangulation gN_{ω_1}, and the covering ω_2' consist of preimages of the mapping f_1 of main stars (see [1], Chapter 3, § 2, Section 3) of the triangulation $(gN_{\omega_1})^*$. Suppose also that a finite component covering ω_2 of the space X inscribed into the covering ω_2', gN_{ω_2} is a standard geometrical realization of the nerve N_{ω_2} and f_2 is a canonical with respect to ω_2 mapping of the space X into $\widetilde{gN}_{\omega_2}$.

Then any mapping $\pi : \widetilde{gN}_{\omega_2} \to \widetilde{gN}_{\omega_1}$ generated by a refinement ω_2 into ω_1 and simplicial with respect to triangulation $\widetilde{gN}_{\omega_2}$ and $(\widetilde{gN}_{\omega_1})^*$ is obtained, according to the Lemma (see [1], Chapter 4, § 1) about the descent with respect to the triangulation $(gN_{\omega_1})^*$ from mapping f_1 (i.e. support arbitrary point $\pi f_2(x)$ is face of support of the point $f_1(x)$ in the triangulation $\left(gN_{\omega_1}\right)^*$).

The proof implies from the case of compact polyhedrons (see [1], Chapter 3, § 1) we turn on to bodies of component of the covering ω_2.

Theorem 1. *Any n-dimensional complete metric superparacompact space X is limit of inverse sequence $S = \left\{\widetilde{K}_i, \pi_i^{i+1}\right\}$, $i = 1, 2, \ldots$, from n-dimensional polyhedrons $\widetilde{K}_i$, being bodies of standard triangulation K_i decomposing to discrete sum of compact polyhedrons; in addition projections π_i^{i+1} are simplicial with respect to K_{i+1} and some triangulation K_i^* of the polyhedron $\widetilde{K}_i$, being subdivision of the triangulation K_i. Every projection $\pi_i : X \to \widetilde{K}_i$ is irreducible with respect to triangulation K_i, $i = 1, 2, \ldots$*

Proof. We construct searching inverse sequence by induction. Let γ_i, $i = 1, 2, \dots$, be $1/2^i$-open covering of the space X. Since $\dim X = n$, then there exists such open covering η of the space X, that any inscribed covering into it has multiplicity $\geq n+1$. By virtue of Proposition 1, to the covering $\{\gamma_1 \wedge \eta\}$ we inscribe irreducible finite component open covering ω_1 of the space X. Nerve of the covering ω_1 we denote by N_1, and as K_1 we denote standard geometrical realization N_1 in Hilbert space R^τ. According to Proposition 2, there exists canonical with respect to ω_1 mapping f_1 of the space X into polyhedron $\widetilde{K}_1$. Because the covering ω_1 is irreducible, then, according to Proposition 3, the mapping f_1 is irreducible mapping with respect to triangulation K_1 and, so, will be mapping onto $\widetilde{K}_1$. The covering ω_1 inscribed into covering $\{\gamma_1 \wedge \eta\}$ of the space X, thus the covering ω_1 has multiplicity $n+1$ and $\dim \widetilde{K}_1 = n$. As the covering ω_1 is finite component polyhedron $\widetilde{K}_1$ is discrete sum of compact polyhedrons. We consider covering φ_1, consisting of preimages main stars of the triangulation K_1^* in the mapping f_1, where K_1^* is such subdivision of the triangulation K_1, that its mesh $< 1/2^2$ (see section a) of Remark 2).

Into covering $\{\varphi_1 \wedge \eta \wedge \gamma_2\}$ we inscribe irreducible finite component open covering ω_2 of the space X. According to Proposition 2 there exists canonical with respect to ω_2 mapping f_2 of the space X into polyhedron $\widetilde{K}_2$, where K_2 is standard geometrical realization of the nerve N_{ω_2} of the covering ω_2 into R^τ. By that reason, that given above, canonical with respect to ω_2 mapping f_2 of the space X into polyhedron $\widetilde{K}_2$ is irreducible with respect to triangulation K_2; the covering ω_2 has multiplicity $n+1$; the polyhedron $\widetilde{K}_2$ is discrete sum of compact polyhedrons and $\dim \widetilde{K}_2 = n$. We take some generated with inscribed ω_2 in $\{\varphi_1 \wedge \eta \wedge \gamma_2\}$ simplicial with respect to the triangulation K_2 and K_1^* mapping $\pi_1^2 : \widetilde{K}_2 \to \widetilde{K}_1$. Then, according to section b) of the Remark 2, the mapping $\pi_1^2 f_2$ is descent of the mapping f_1 with respect to triangulation K_1^*. Therefore $d\left(f_1, \pi_1^2 f_2\right) < 1/2^2$.

Suppose, that for all $i < m$ we constructed: a) n-dimensional polyhedrons $\widetilde{K}_i$, being bodies of standard geometrical realizations in R^τ of nerves N_{ω_i} of irreducible finite component coverings ω_i of the space X, inscribed into coverings $\{\eta \wedge \gamma_i\}$, $i = 1, 2, \dots$; b) canonical with respect to ω_i mappings $f_i : X \to \widetilde{K}_i$, being irreducible mappings with respect to triangulations K_i; c) mappings $\pi_{i-1}^i : \widetilde{K}_i \to \widetilde{K}_{i-1}$, $2 < i < m$, which simplicial with respect to triangulation K_i and some triangulation K_{i-1}^* of polyhedron $\widetilde{K}_{i-1}$, being subdivision of triangulation K_{i-1}; in this connection the mapping $\pi_{i-1}^i f_i$ is obtained from f_{i-1} by descent with respect to K_{i-1}^*; d) mappings $\pi_j^i = \pi_j^{j+1} \dots \pi_{i-1}^i$, π_i^i, $j < i$, satisfy inequalities $d\left(\pi_j^{i-1} f_{i-1}, \pi_j^i f_i\right) < 1/2^i$.

Assume now $i = m$. According to the Remark 1, the polyhedron $\widetilde{K}_{m-1}$ is discrete sum of compact polyhedrons $\widetilde{K}_{m-1}^\beta$, $\beta \in L$, being bodies of standard realizations K_{m-1}^β into R^τ of nerves of components of the covering ω_{m-1}. In triangulation K_{m-1}^β, $\beta \in L$, there exists such barycentric subdivision $\left(K_{m-1}^\beta\right)^{s(\beta)}$, that all simlexes of the triangulation $\left(K_{m-1}^\beta\right)^{s(\beta)}$ and their images into polyhedrons $\widetilde{K}_j$ in the mapping π_j^{m-1}, $j \leq i \leq m-2$,, have diameters $< 1/2^m$. Suppose $(K_{m-1})^*$ coinciding with $\left(K_{m-1}^\beta\right)^{s(\beta)}$ on $\left(\widetilde{K_{m-1}^\beta}\right)$. Clearly, that all simplexes of the triangulation $(K_{m-1})^*$ and their images into polyhedrons $\widetilde{K}_j$ in mappings π_j^{m-1}, $j \leq i \leq m-2$,, have diameters $< 1/2^m$. Into the covering $\{\varphi_{m-1} \wedge \eta \wedge \gamma_m\}$, where φ_{m-1} consists on preimages of main stars of the triangulation K_{m-1}^*, in the mapping f_{m-1}, according to Proposition 1, we inscribe irreducible finite component open covering ω_m of the space X. There exists canonical with respect to ω_m mapping of the space X into polyhedron $\widetilde{K}_m$, where K_m is standard geometrical realization of the nerve N_{ω_m} of the covering ω_m into R^τ. As before, canonical with respect to ω_m mapping f_m of the space X into polyhedron $\widetilde{K}_m$ is irreducible with respect to triangulation K_m (and, so, will be mapping onto $\widetilde{K}_m$); the covering ω_m have multiplicity $n+1$; polyhedron $\widetilde{K}_m$ is discrete sum of compact polyhedrons and $\dim \widetilde{K}_m = n$. We take some mapping $\pi^m_{m-1} : \widetilde{K}_m \to \widetilde{K}_{m-1}$ generated by ω_m inscribed into $\{\varphi_{m-1} \wedge \eta \wedge \gamma_m\}$ simplicial with respect to triangulation K_m and K^*_{m-1}.

Then, according to section b) of the Remark 2, the mapping $\pi^m_{m-1} f_m$ is descent of mapping f_{m-1} with respect to triangulation K^*_{m-1}. Therefore

$$d\left(f_{m-1}, \pi^m_{m-1} f_m\right) < \frac{1}{2^m}, \quad d\left(\pi^{m-1}_j f_{m-1}, \pi^m_j f_m\right) < \frac{1}{2^m}, \quad j < m-1. \tag{1}$$

Continuing construction n-dimensional polyhedrons $\widetilde{K}_i$ and mappings π^{i+1}_i, we obtain inverse sequence $\dot{s} = \left\{\widetilde{K}_i, \pi^{i+1}_i\right\}$, $i = 1, 2, \ldots$, satisfying all conditions of theorem. By $\widetilde{s}$ we denote limit of inverse sequence s. Consider for each $i = 1, 2, 3, \ldots$ the sequence of mappings

$$f_i,\ \pi^{i+1}_i f_{i+1},\ \pi^{i+2}_i f_{i+2},\ \ldots \tag{2}$$

of the space X into polyhedron $\widetilde{K}_i$. The proof of that fact, which all later mappings of the sequence (2) are obtained from f_i by descent with respect to triangulation K_i, similarly compact case of the space X (see [1], Chapter 5, § 5, Freudenthal's Theorem). According to second inequality of (1) we have

$$d\left(\pi^{m-1}_i f_{m-1}, \pi^m_i f_m\right) < \frac{1}{2^m}.$$

Therefore for any point $x \in X$ the sequence $\{\pi^m_i f_m(x)\}$, $m = i+1, i+2, \ldots$, is fundamental sequence. Since the polyhedron $\widetilde{K}_i$ is complete metrizable, then the sequence $\{\pi^m_i f_m(x)\}$, $m = i+1, \ldots$, is convergent at some point $g_i(x) \in \widetilde{K}_i$. Sequence of mappings $\{\pi^m_i f_m\}$, $m = i+1, i+2, \ldots$, is convergent to g_i uniformly, therefore mapping $g_i : X \to \widetilde{K}_i$ is continuous. Since all mappings $\pi^m_i f_m$ are obtained f_i by descent with respect to triangulation K_i, then the mapping g_i also has this property (see [1], Chapter 4, § 1, Lemma 2). Therefore mappings $g_i : X \to \widetilde{K}_i$, $i = 1, 2, \ldots$, are canonical mappings with respect to covering ω_i.

Furthermore, according to Proposition 3, mappings $g_i : X \to \widetilde{K}_i$, $i = 1, 2, \ldots$, are irreducible mappings with respect to triangulation K_i(and, so, will be mappings on $\widetilde{K}_i$). The relation $g_i = \pi^j_i g_j$ when $i < j$ is checked by standard way (see [1], Chapter 5, § 5).

Since each mapping g_i is ω_i-mapping of the space X into polyhedron $\widetilde{K}_i$, and system of open coverings ω_i, $i = 1, 2, \ldots$, of the space X is refinement (see [1], Chapter 1, § 7, Definition 10) (since the covering ω_i is inscribed in γ_i), then limit $g : X \to \widetilde{S} \subseteq \prod_{i=1}^{\infty} \widetilde{K}_i$ of mappings g_i is (see [1], Chapter 6, § 4, Lemma 2) embedding of the space X into limit $\widetilde{S}$ of inverse sequence S. We prove, that g there is mapping of the space X on limit $\widetilde{S}$ of the inverse sequence S.

We take some point $y^0 \in \widetilde{S}$ and assume $y^0 = \left\{y^0_i, i = 1, 2, \ldots\right\}$. Consider closed sets $\Phi_i = g_i^{-1} y^0_i$, $i = 1, 2, \ldots$ in X. Since g_i is ω_i-mapping, then $\Phi_i \subseteq O_{\alpha(i)} \in \omega_i$, $i = 1, 2, \ldots$ We prove, that $\Phi_{i+1} \subseteq \Phi_i$, $i = 1, 2, \ldots$

Since $y^0_i = \pi^{i+1}_i y^0_{i+1}$, then

$$y^0_{i+1} \subseteq \left(\pi^{i+1}_i\right)^{-1} y^0_i, \quad i = 1, 2, \ldots \tag{$*$}$$

Then from inclusion $(*)$ and the equality $g_i = \pi^{i+1}_i g_{i+1}$ follows that

$$g^{-1}_{i+1} y^0_{i+1} = \Phi_{i+1} \subseteq g^{-1}_{i+1} \left(\pi^{i+1}_i\right)^{-1} y^0_i = g^{-1}_i y^0_i = \Phi_i, \quad i = 1, 2, \ldots$$

So, the system $\{\Phi_i, i = 1, 2, \ldots\}$ closed in X sets Φ_i, the sets which diameters tends to zero, is embedded. Then from completeness of the space X follows that intersection of the sets Φ_i nonempty and consists on one point. Suppose $\bigcap_{i=1}^{\infty} \Phi_i = \{x^0\}$. Since $g_i\Phi_i = y_i^0$ and $x^0 \in \Phi_i$, then $g_i(x^0) = y_i^0$, $i = 1, 2, \ldots$ Consequently, $gx^0 = y^0$ and therefore $y^0 \in gX$. Since y^0 is any point of the space $\widetilde{S}$, then from here follows, that g is (topological) mapping of the space X onto limit $\widetilde{S}$ of the inverse sequence S.

Note that in identification of points $x \in X$ and $gx \in \widetilde{S}$ projections $\pi_i : \widetilde{S} \to \widetilde{K}_i$ are identified with irreducible with respect to triangulation K_i mappings g_i. □

This theorem is generalization of the Freudenthal's theorem [5].

Corollary 1. *Any n-dimensional metric superparacompact space X is homeomorphic to the everywhere dense subset of the limit $\widetilde{S}$ of the inverse sequence $S = \left\{\widetilde{K}_i, \pi_i^{i+1}\right\}$, $i = 1, 2, \ldots$, from n-dimensional polyhedron $\widetilde{K}_i$, being bodies of standard triangulation K_i and decomposing into discrete sum of compact polyhedrons; in addition projections π_i^{i+1} are simplicial with respect to K_{i+1} and some triangulation K_i^* of the polyhedron $\widetilde{K}_i$, being subdivision of the triangulation K_i. Each projection $\pi_i : X \to \widetilde{K}_i$ is irreducible with respect to the triangulation K_i, $i = 1, 2, \ldots$*

Proposition 4. *Any superparacompact complete with respect to Cech ($p-$) space X [6] is perfectly mapped into Baire space $B(\tau)$ of the weight τ (onto 0-dimensional in the sense* dim *metrizable space of the weight $\leq \tau$).*

Proof. The space X is perfectly mapped (see [7], Theorem 2) onto 0-dimensional in the sense dim complete metrizable (metrizable) space X_0. Therefore $\omega X_0 \leq \tau$. Since any 0-dimensional in the sense dim complete metrizable space of the weight $\leq \tau$ is homeomorphic (see [8], Proposition 5.1) closed subspace of generalized Baire space $B(\tau)$ of the weight τ and composition perfect mappings are perfect, then hence follows, that the space X is perfectly mapped into Baire space $B(\tau)$ of the weight τ (onto 0-dimensional in the sense dim metrizable space of the weight $\leq \tau$). □

Corollary 2. *Any superparacompact complete metrizable space X of the weight $\leq \tau$ is perfectly mapped into Baire space $B(\tau)$ of the weight τ.*

Theorem 2. *For metrizable space X following statements are equivalent: a) X is superparacompact complete metrizable space of weight $\leq \tau$; b) X is perfectly mapping into Baire space $B(\tau)$ of the weight τ; c) X is closed included into product $B(\tau) \times Q^\infty$ of Baire space $B(\tau)$ of the weight τ on Hilbert cub Q^∞.*

Proof. If in the condition of the theorem $\tau < \aleph_0$, then all statements of the theorem are evidently. Therefore we consider the case, when $\tau \geq \aleph_0$.

The statement b) implies from statement a) because of Proposition 4.

The case b) $\Rightarrow$ c). Let f be perfect mapping of the space X into Baire space $B(\tau)$ of the weight τ. There exists (see [9, theorem 3]) such embedding $g : X \to B(\tau) \times Q^\infty$, that $f = \pi \circ g$, where π is the projection $B(\tau) \times Q^\infty$ onto $B(\tau)$. Since the mapping f is perfect, and the space $B(\tau) \times Q^\infty$ is Hausdorff space, then the mapping g is perfect [4]. Thus, g is closed embedding of the space X into product $B(\tau) \times Q^\infty$ of Baire space $B(\tau)$ of the weight τ to Hilbert cub Q^∞.

Now we derive from statement c) the statement a). The product $B(\tau) \times Q^\infty$ of Baire space $B(\tau)$ of the weight τ to Hilbert cub Q^∞ is superparacompact (see [3], Corollary 1). It is known, that the product $B(\tau) \times Q^\infty$ is complete metrizable and $w\left(B(\tau) \times Q^\infty\right) = \tau$. Then from monotonicity of complete metrizable and superparacompact (see [3]) by closed subspaces follows, that the space X is superparacompact and complete metrizable. Since $w\left(B(\tau) \times Q^\infty\right) = \tau$, then $wX \leq \tau$. □

We note, that Theorem 2 is extension of the theorem Morita [10] about universality of the product $B(\tau) \times Q^\infty$ in the class of all strongly metrizable space of the weight $\leq \tau$.

Theorem 3. *For Hausdorff space X following statement are equivalent: a) X is superparacompact (complete) metrizable space of the weight $\leq \tau$ and $\dim X \leq n$; b) X is closed embedded into product (Baire space $B(\tau)$ of the weight τ) of 0-dimensional in the sense* dim *of metrizable space of the weight τ onto universal n-dimensional compact Φ^n.*

Proof. By virtue of Proposition 4, the space X is perfectly mapped (into Baire space $B(\tau)$ of the weight τ) onto 0-dimensional in the sense dim metrizable space X_0 of the weight $\leq \tau$.

Since the space X is strongly metrizable, $\dim X \leq n$ and $wX \leq \tau$, then by virtue of Nagata's theorem (see [11]), the space X is topological mapped into product $B(\tau) \times \Phi^n$ of generalized Baire space $B(\tau)$ of the weight τ to universal n-dimensional compact Φ^n. Then the space X is homomorphic (see [12], Proposition 59, Chapter VI, § 2) to closed subspace of the product $(B(\tau) \times B(\tau) \times \Phi^n)\, X_0 \times B(\tau) \times \Phi^n$. Suppose $(B(\tau) = B(\tau) \times B(\tau))\ R^0_\tau = X_0 \times B(\tau)$. The space $(B(\tau))R^0_\tau$ (complete) metrizable, $(wB(\tau) = \tau)$ $wR^0_\tau = \tau$ and 0-dimensional in the sense dim [4].

We deduce from statement b) the statement a). The product $(B(\tau) \times \Phi^n)\ R^0_\tau \times \Phi^n$ is superparacompact (see [3], Corollary 1). It is known [1], that the product $(B(\tau) \times \Phi^n)\ R^0_\tau \times \Phi^n$ (complete) metrizable, $(\dim (B(\tau) \times \Phi^n) = n)$ $\dim \left(R^0_\tau \times \Phi^n\right) = n$ and $(w\,(B(\tau) \times \Phi^n) = \tau)$ $w\left(R^0_\tau \times \Phi^n\right) = \tau$, then from monotonicity of the superparacompact property (see [3]), complete metrizability of dimensionality dim by closed subspaces follows, that the space X is superparacompact, (complete) metrizable and $\dim X \leq n$. Clearly, that and $wX \leq \tau$. □

Theorem 3 is expansion of the Nagata's theorem [11] about embedding n-dimensional strongly metrizable space in $B(\tau) \times \Phi^n$ to the case superparacompact.

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