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ON UNIVERSAL COORDINATE TRANSFORM IN KINEMATIC CHANGEABLE SETS

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This paper is dedicated to the memory of professor Myroslav Gorbachuk

Abstract. This work is devoted to a study of abstract coordinate transforms in kinematic changeable sets. Investigations in this direction may be interesting for astrophysics, because there exists a hypothesis that, in a large scale of the Universe, physical laws (in particular, the laws of kinematics) may be different from the laws acting in a neighborhood of our solar System.

1. Introduction

Due to the OPERA experiments conducted within 2011-2012 years [1], quite a lot of physical works have appeared where the authors are trying to modify the special relativity theory to make its conclusions agree with the hypothesis of existence of objects moving at velocity greater than the velocity of light. Despite the fact that exceeding the velocity of light by neutrinos in OPERA experiments (2011-2012) were not confirmed later, the problem of constructing a theory of super-light movement (which was posed in the papers [6, 7]) remains actual for more than 50 last years. At the present time a few of different kinematic theories of tachyon motion exist. Therefore, it arises a problem of constructing a new mathematical structures that would allow to simulate the evolution of physical systems in a framework of different laws of kinematics. Due to the lack of experimental verification of conclusions of tachyon kinematics theories, such mathematical structures may at least guarantee the correctness of obtaining these conclusions in accordance with the postulates of these theories.

The problem formulated in the previous paragraph is closely connected with the famous sixth Hilbert problem about mathematically strict formulation of the foundations of theoretical physics. A lot of papers were devoted to this problem (for example, see [2, 12, 13, 33–35, 38, 41–43]), but completely it is not solved to this day. In connection with the sixth Hilbert problem in the papers [18–20, 22, 23, 25, 29] a theory of changeable sets is constructed. From an intuitive point of view, changeable sets are sets of objects which, unlike elements of ordinary (static) sets may be in the process of continuous transformations, and which may change properties depending on the point of view on them (that is depending on the reference frame).

At a physical level, the problem of investigating the kinematics with arbitrary space-time coordinate transforms for inertial reference frames was presented in the [3] for the case, where the space of geometric variables is three-dimensional and Euclidean. The particular cases of coordinate transforms considered in [3] are the (three-dimensional) classical Lorentz transforms as well as the generalized Lorentz transforms (in the sense of E. Recami and V. Olkhovsky) for reference frames moving at a velocity greater than the velocity of light [16, 30, 39, 40, 46]. In the papers [17, 21] the general definition of linear

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coordinate transforms and generalized Lorentz transforms is given in the case where the space of geometric variables is any real Hilbert space.

It should be noted that the mathematical apparatus of the papers [3, 16, 17, 21, 30, 39, 40, 46] is not based on the theory of changeable sets, which greatly reduces its generality. In particular, mathematical apparatus of these papers allows only to study universal coordinate transforms (that is the coordinate transforms under which the geometrically-time provision of an arbitrary material object in any reference frame is determined by the geometrically-time position of this object in a certain, fixed frame, independently of any internal properties of the object). In the papers [24, 26–29], based on the theory of changeable sets, the theory of kinematic changeable sets is constructed and strict definitions for the notions of coordinate transform and universal coordinate transform are introduced. In the present paper, based on the results of [24, 26–29], we prove that, in the classical Galilean and the Lorentz-Poincare kinematics, the universal coordinate transform always exists. Also we construct one class of kinematics in which every particle can have its own "velocity of light" and prove that, in these kinematics, a universal coordinate transform does not exist in nontrivial cases.

2. Basic concepts of the theories of changeable and kinematic changeable sets

In the present section we recall necessary denotations and concepts of the theories of changeable sets and kinematic changeable sets, introduced in [18, 24] (see also [19, 20, 22, 23, 25–29]). The most complete and detailed explanation of the theories of changeable sets and kinematic changeable sets can be found in the preprint [29].

2.1. **Base changeable sets.** Base changeable sets may be interpreted as mathematical abstractions of models of physical and other processes of macrocosm in the framework of one, fixed, frame of reference.

Let $\mathbb{T} = (\mathbf{T}, \leq)$ be any linearly (totally) ordered set (the sense of [8, p. 12]) and let $\mathcal{X}$ be any nonempty set. For any ordered pair $\omega = (t, x) \in \mathbf{T} \times \mathcal{X}$ we use the following notations:

$$\mathsf{bs}\,(\omega) := x, \quad \mathsf{tm}\,(\omega) := t.$$

Recall [25] [1] that an ordered triple of the kind $\mathcal{B} = (\mathbf{B}, \mathbb{T}, \Leftarrow)$, where $\mathbf{B} \subseteq \mathbf{T} \times \mathcal{X}$, is called a ***base changeable set***[2] if and only if the following conditions are satisfied:

(1) $\mathbf{B} \neq \emptyset$ and $\Leftarrow$ is a reflexive binary relation on $\mathbf{B}$ (that is $\forall \omega \in \mathbf{B}\ \omega \Leftarrow \omega$);
(2) for arbitrary $\omega, \omega_2 \in \mathbf{B}$, the conditions $\omega_2 \Leftarrow \omega_1$ and $\omega_1 \neq \omega_2$ imply the inequality $\mathsf{tm}\,(\omega_1) < \mathsf{tm}\,(\omega_2)$, where $<$ is the strict order relation generated by the non-strict order $\leq$ of linearly ordered set $\mathbb{T} = (\mathbf{T}, \leq)$.

Remark 1. For an arbitrary base changeable set $\mathcal{B} = (\mathbf{B}, \mathbb{T}, \Leftarrow) = (\mathbf{B}, (\mathbf{T}, \leq), \Leftarrow)$ (where $\mathbf{B} \subseteq \mathbf{T} \times \mathcal{X}$) we use the following *notations and terminology*:

$$\mathbb{B}\mathfrak{s}(\mathcal{B}) := \mathbf{B}; \qquad \underset{\mathcal{B}}{\leftarrow} := \Leftarrow;$$

$$\mathbb{T}\mathbf{m}(\mathcal{B}) := \mathbb{T}; \qquad \mathbf{Tm}(\mathcal{B}) := \mathbf{T};$$

$$\mathfrak{Bs}(\mathcal{B}) := \{x \in \mathcal{X} \mid \exists\, \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\ (\mathsf{bs}\,(\omega) = x)\} = \{\mathsf{bs}\,(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}\,. \tag{1}$$

- The set $\mathfrak{Bs}(\mathcal{B})$ is called a basic set or a set of all elementary states of $\mathcal{B}$.

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[1] In the papers [18, 22, 29] we gave a something different from [25], but logically equivalent, definition of the concept of a base changeable set, which is less laconic and requiring an introduction of additional notions.

[2] Note that in some early works (for example in [18]) the term "basic changeable set" is used instead of the term "base changeable set". This is due to existence of two variants of translation of this term from Ukrainian language.

- The set $\mathbb{Bs}(\mathcal{B})$ is called a set of all elementary-time states of $\mathcal{B}$.
- The set $\mathbf{Tm}(\mathcal{B})$ is called a set of time points of $\mathcal{B}$.
- The relation $\underset{\mathcal{B}}{\leftarrow}$ is called a base of elementary processes of $\mathcal{B}$.

Remark 2. In the cases where the base changeable set $\mathcal{B}$ is known in advance we use the notation $\leftarrow$ instead of the notation $\underset{\mathcal{B}}{\leftarrow}$.

2.2. **Changeable sets.** Changeable sets to be introduced in this subsection may be interpreted as abstractions of models of physical and other processes of macrocosm in the framework of observation in many, different, reference frames.

Definition 1. *Let $\overleftarrow{\mathcal{B}} = (\mathcal{B}_\alpha \mid \alpha \in \mathcal{A})$ be any indexed family of base changeable sets (where $\mathcal{A} \neq \emptyset$ is some set of indexes). A system of mappings $\overleftarrow{\mathfrak{U}} = (\mathfrak{U}_{\beta\alpha} \mid \alpha, \beta \in \mathcal{A})$ of the kind*

$$\mathfrak{U}_{\beta\alpha} : 2^{\mathbb{Bs}(\mathcal{B}_\alpha)} \longmapsto 2^{\mathbb{Bs}(\mathcal{B}_\beta)} \quad (\alpha, \beta \in \mathcal{A})$$

*is referred to as **unification of perception** on $\overleftarrow{\mathcal{B}}$ if and only if the following conditions are satisfied:*

(1) *$\mathfrak{U}_{\alpha\alpha} A = A$ for any $\alpha \in \mathcal{A}$ and $A \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$.*
(Here and in the sequel, we denote by $\mathfrak{U}_{\beta\alpha} A$ the action of the mapping $\mathfrak{U}_{\beta\alpha}$ to the set $A \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$, that is $\mathfrak{U}_{\beta\alpha} A := \mathfrak{U}_{\beta\alpha}(A)$.)
(2) *Any mapping $\mathfrak{U}_{\beta\alpha}$ is a monotonous mapping of sets, i.e., for any $\alpha, \beta \in \mathcal{A}$ and $A, B \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$ the condition $A \subseteq B$ assures $\mathfrak{U}_{\beta\alpha} A \subseteq \mathfrak{U}_{\beta\alpha} B$.*
(3) *For any $\alpha, \beta, \gamma \in \mathcal{A}$ and $A \subseteq \mathbb{Bs}(\mathcal{B}_\alpha)$ the following inclusion holds:*

$$\mathfrak{U}_{\gamma\beta}\mathfrak{U}_{\beta\alpha} A \subseteq \mathfrak{U}_{\gamma\alpha} A.$$

*In this case we call the mappings $\mathfrak{U}_{\beta\alpha}$ $(\alpha, \beta \in \mathcal{A})$ **unification mappings**, and a triple of the kind*

$$\mathcal{Z} = \left(\mathcal{A}, \overleftarrow{\mathcal{B}}, \overleftarrow{\mathfrak{U}}\right)$$

*is called a **changeable set**.*

Remark 3 (on notations). Assume that $\mathcal{Z} = \left(\mathcal{A}, \overleftarrow{\mathcal{B}}, \overleftarrow{\mathfrak{U}}\right)$ is a changeable set, where $\overleftarrow{\mathcal{B}} = (\mathcal{B}_\alpha \mid \alpha \in \mathcal{A})$ is an indexed family of base changeable sets and $\overleftarrow{\mathfrak{U}} = (\mathfrak{U}_{\beta\alpha} \mid \alpha, \beta \in \mathcal{A})$ is a unification of perception on $\overleftarrow{\mathcal{B}}$. Further we will use the following terms and notations:

1) The set $\mathcal{A}$ will be called an ***index set*** of the changeable set $\mathcal{Z}$, and it will be denoted by $\mathcal{I}nd\,(\mathcal{Z})$.

2) For any index $\alpha \in \mathcal{I}nd\,(\mathcal{Z})$ the pair $(\alpha, \mathcal{B}_\alpha)$ will be referred to as a ***reference frame*** of the changeable set $\mathcal{Z}$.

3) The set of all reference frames of $\mathcal{Z}$ will be denoted by $\mathcal{L}k\,(\mathcal{Z})$

$$\mathcal{L}k\,(\mathcal{Z}) := \{(\alpha, \mathcal{B}_\alpha) \mid \alpha \in \mathcal{I}nd\,(\mathcal{Z})\}\,.$$

Typically, reference frames will be denoted by small Gothic letters ($\mathfrak{l}, \mathfrak{m}, \mathfrak{k}, \mathfrak{p}$ and so on).

4) For $\mathfrak{l} = (\alpha, \mathcal{B}_\alpha) \in \mathcal{L}k\,(\mathcal{Z})$ we introduce the following notations:

$$\mathtt{ind}\,(\mathfrak{l}) := \alpha; \quad \mathfrak{l}^{\hat{}} := \mathcal{B}_\alpha.$$

Thus, for any reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{Z})$ an object $\mathfrak{l}^{\hat{}}$ is a base changeable set. Further, when it does not cause confusion, for any reference frame $\mathfrak{l} \in \mathcal{L}k\,(\mathcal{Z})$ the symbol "$\hat{}$" will be omitted in the notations $\mathfrak{Bs}\,(\mathfrak{l}^{\hat{}})$, $\mathbb{Bs}\,(\mathfrak{l}^{\hat{}})$, $\mathbf{Tm}\,(\mathfrak{l}^{\hat{}})$, $\mathbb{Tm}\,(\mathfrak{l}^{\hat{}})$, $\underset{\mathfrak{l}^{\hat{}}}{\leftarrow}$, and the notations $\mathfrak{Bs}\,(\mathfrak{l})\,,\ \mathbb{Bs}\,(\mathfrak{l})\,,\ \mathbf{Tm}\,(\mathfrak{l})\,,\ \mathbb{Tm}\,(\mathfrak{l})\,,\ \underset{\mathfrak{l}}{\leftarrow}$ will be used instead.

5) For any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ the mapping $\mathfrak{U}_{\mathtt{ind}(\mathfrak{m}),\mathtt{ind}(\mathfrak{l})}$ will be denoted by $\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle$. Hence

$$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle = \mathfrak{U}_{\mathtt{ind}(\mathfrak{m}),\mathtt{ind}(\mathfrak{l})}.$$

In the case where the changeable $\mathcal{Z}$ set is known in advance, the symbol $\mathcal{Z}$ in the above notations will be omitted, and the notation "$\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle$" will be used instead.

6) In the case where it does not cause confusion, we will use the notation $\leftarrow$ instead of the notation $\underset{\mathfrak{l}}{\leftarrow}$.

7) For any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$, we reserve the terminology introduced in Remark 1 (where the symbol $\mathcal{B}$ should be replaced with the symbol "$\mathfrak{l}$" and the phrase "base changeable set" should be replaced with the phrase "reference frame").

Definition 2. *We say, that a changeable set $\mathcal{Z}$ is* ***precisely visible*** *if and only if for any reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ and for any element $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ there exist a unique element $\omega' \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ such that $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\omega'\}$.* [3]

Let $\mathcal{Z}$ be any precisely visible changeable set and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathcal{Z})$ be any reference frames of $\mathcal{Z}$. For any $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ we denote by $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle \omega$ (or by $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega$) a unique (in accordance with Definition 2) element $\omega' \in \mathbb{B}\mathfrak{s}(\mathfrak{m})$ such that $\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\omega'\}$. Hence, we have $\forall \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\ \langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle \{\omega\} = \{\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle \omega\}$. The mapping $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle : \mathbb{B}\mathfrak{s}(\mathfrak{l}) \mapsto \mathbb{B}\mathfrak{s}(\mathfrak{m})$ will be called a ***precise unification mapping*** of $\mathcal{Z}$.

2.3. Kinematic changeable sets.

Kinematic changeable sets are mathematical objects, in which changeable sets are equipped with different geometrical or topological structures, namely metric, topological, linear, Banach, Hilbert and other spaces. Such mathematical objects may be used for constructing models of physical and other processes of macrocosm, acting in the framework of some space environment and including the spatial movement of bodies. For simplicity we restrict our consideration to the case where the geometrical environment of changeable set is generated by a linear normed space [4]. Moreover, here we consider only kinematic changeable sets with constant (unchanging over time) geometry.

Definition 3. *Let $\mathcal{Z}$ be any changeable set. An indexed family of type*

$$\mathcal{G} = \left(\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}, k_{\mathfrak{l}} \right) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z}) \right)$$

will be called a geometric environment of the changeable set $\mathcal{Z}$, if and only if for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathcal{Z})$ the following conditions are satisfied:

1. $\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})} \right)$ *is a linear normed space over the real field $\mathbb{R}$ or the complex field $\mathbb{C}$.*
2. $k_{\mathfrak{l}} : \mathfrak{B}\mathfrak{s}(\mathfrak{l}) \mapsto \mathfrak{X}_{\mathfrak{l}}$ *is a mapping from $\mathfrak{B}\mathfrak{s}(\mathfrak{l})$ to $\mathfrak{X}_{\mathfrak{l}}$.*

In this context a pair $\mathfrak{C} = (\mathcal{Z}, \mathcal{G}) = \left(\mathcal{Z}, \left(\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}, k_{\mathfrak{l}} \right) \mid \mathfrak{l} \in \mathcal{L}k(\mathcal{Z}) \right) \right)$ will be called a ***vector kinematic changeable set****. Taking into account that we consider only vector kinematic changeable sets in this article, we further use the terms "****kinematic changeable set****" or "****kinematic set****" instead of "vector kinematic changeable set".*

We say that a kinematic set $\mathfrak{C} = (\mathcal{Z}, \mathcal{G})$ is ***precisely visible*** *if and only if the changeable set $\mathcal{Z}$ is precisely visible.*

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[3] In some papers (see [20, Definition 3.3], [29, Definition I.12.3]) it had been given another, different, definition of the notion of a precisely visible changeable set. Using [29, Corollary I.12.5 and Assertion I.12.11] it can be proved that Definition 2 is equivalent to the mentioned definitions, given in [20, 29].

[4] More general variants of geometrical environments of changeable sets are considered in the papers [24, 26–29]. But the accepted restrictions are quite sufficient for obtaining the main results of this work.

Let, $\mathfrak{C} = \left(\mathcal{Z}, \left(\left(\mathfrak{X}_{\mathfrak{l}}, \|\cdot\|_{(\mathfrak{l})}, k_{\mathfrak{l}}\right) \mid \mathfrak{l} \in \mathcal{L}k\left(\mathcal{Z}\right)\right)\right)$ be any kinematic set. The sets

$$\mathcal{L}k\left(\mathfrak{C}\right) := \mathcal{L}k\left(\mathcal{Z}\right); \quad \mathcal{I}nd\left(\mathfrak{C}\right) := \mathcal{I}nd\left(\mathcal{Z}\right)$$

will be called a set of all reference frames and a set of indexes of the kinematic set $\mathfrak{C}$ (correspondingly).

We further use the following notations for arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\left(\mathfrak{C}\right) = \mathcal{L}k\left(\mathcal{Z}\right)$:

1: We keep all notations, introduced for reference frames of changeable sets (namely $\mathtt{ind}(\mathfrak{l})$, $\mathfrak{l}^{\wedge}$, $\mathfrak{B}\mathfrak{s}(\mathfrak{l})$, $\mathbb{B}\mathfrak{s}(\mathfrak{l})$, $\underset{\mathfrak{l}}{\leftarrow}$, $\mathbf{Tm}(\mathfrak{l})$, $\mathbb{T}\mathbf{m}(\mathfrak{l})$) together with abbreviated variants of these notations introduced in item 6) of Remark 3 and the terminology described in item 7) of Remark 3 (where the symbol "$\mathcal{Z}$" should be replaced with "$\mathfrak{C}$").

2: For unification mappings we use the following notation:

$$\langle \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C}\rangle := \langle \mathfrak{m} \leftarrow \mathfrak{l},\, \mathcal{Z}\rangle\,,$$

and, in the case of precisely visible kinematic set $\mathfrak{C}$, we use the notation

$$\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathfrak{C}\rangle\, \omega := \langle !\, \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z}\rangle\, \omega \quad \left(\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right).$$

3: We denote $\mathbf{Zk}(\mathfrak{l};\, \mathfrak{C}) := \mathfrak{X}_{\mathfrak{l}}$, $\|\cdot\|_{\mathfrak{l},\mathfrak{C}} := \|\cdot\|_{(\mathfrak{l})}$, $\mathsf{q}_{\mathfrak{l}}\left(x, \mathfrak{C}\right) := k_{\mathfrak{l}}(x) \in \mathfrak{X}_{\mathfrak{l}} = \mathbf{Zk}(\mathfrak{l};\, \mathfrak{C})$ $\left(x \in \mathfrak{B}\mathfrak{s}\left(\mathfrak{l}\right)\right)$.

The set $\mathbf{Zk}(\mathfrak{l};\, \mathfrak{C})$ will be called a set of ***coordinate values*** for the reference frame $\mathfrak{l}$ in the kinematic set $\mathfrak{C}$.

4: In the cases where the kinematic set $\mathfrak{C}$ is known in advance, we will use the **abbreviated variants of notations** $\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle$, $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega$, $\mathbf{Zk}(\mathfrak{l})$, $\|\cdot\|_{\mathfrak{l}}$ and $\mathsf{q}_{\mathfrak{l}}\left(x\right)$ instead of $\langle \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C}\rangle$, $\langle !\, \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C}\rangle\, \omega$, $\mathbf{Zk}(\mathfrak{l};\, \mathfrak{C})$, $\|\cdot\|_{\mathfrak{l},\mathfrak{C}}$ and $\mathsf{q}_{\mathfrak{l}}\left(x, \mathfrak{C}\right)$ (correspondingly).

3. Coordinate transforms in kinematic sets

Let, $\mathfrak{C}$ be any kinematic set. For any reference frame $\mathfrak{l} \in \mathcal{L}k\left(\mathfrak{C}\right)$ we introduce the following notations:

$$\mathbb{M}k(\mathfrak{l}; \mathfrak{C}) := \mathbf{Tm}(\mathfrak{l}) \times \mathbf{Zk}(\mathfrak{l}; \mathfrak{C}).$$

$$\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega; \mathfrak{C}) := \left(\mathsf{tm}\left(\omega\right), \mathsf{q}_{\mathfrak{l}}(\mathsf{bs}\left(\omega\right);\, \mathfrak{C})\right) \in \mathbb{M}k(\mathfrak{l}; \mathfrak{C}), \quad \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}).$$

The set $\mathbb{M}k(\mathfrak{l}; \mathfrak{C})$ will be called a by ***Minkowski set*** of the reference frame $\mathfrak{l}$ in the kinematic set $\mathfrak{C}$. The value $\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega; \mathfrak{C})$ will be called ***Minkowski coordinates*** of the elementary-time state $\omega \in \mathbb{B}\mathfrak{s}\left(\mathfrak{l}\right)$ ***in the reference frame*** $\mathfrak{l}$.

In the cases where the kinematic set $\mathfrak{C}$ is known in advance, we use the notations $\mathbb{M}k(\mathfrak{l})$, $\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega)$ instead of the notations $\mathbb{M}k(\mathfrak{l}; \mathfrak{C})$, $\mathbf{Q}^{\langle \mathfrak{l}\rangle}(\omega; \mathfrak{C})$ (correspondingly).

Definition 4. *Let $\mathfrak{C}$ be any precisely visible kinematic set and $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k\left(\mathfrak{C}\right)$ be arbitrary reference frames of $\mathfrak{C}$.*

(1) *The mapping $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle}\left(\,\cdot\,; \mathfrak{C}\right) : \mathbb{B}\mathfrak{s}(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$, represented by the formula*

$$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle}(\omega; \mathfrak{C}) = \mathbf{Q}^{\langle \mathfrak{m}\rangle}\left(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l}\rangle\, \omega\right), \quad \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}),$$

*will be called an **actual coordinate transform** from $\mathfrak{l}$ to $\mathfrak{m}$.*
For any $\omega \in \mathbb{B}\mathfrak{s}\left(\mathfrak{l}\right)$, the value $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l}\rangle}(\omega; \mathfrak{C})$ may be interpreted as Minkowski coordinates of the elementary-time state ω in the (another) reference frame $\mathfrak{m} \in \mathcal{L}k\left(\mathfrak{C}\right)$.

(2) *The mapping $\widetilde{Q} : \mathbb{M}k(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$ will be called a **universal coordinate transform** from $\mathfrak{l}$ to $\mathfrak{m}$ if and only if the following holds:*
   - *$\widetilde{Q}$ is a bijection (a one-to-one mapping) between $\mathbb{M}k(\mathfrak{l})$ and $\mathbb{M}k(\mathfrak{m})$.*

- *For any elementary-time state $\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$, the following equality holds true:*

$$\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C}) = \widetilde{Q}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)\right).$$

(3) *We say that reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ **allow a universal coordinate transform** if and only if at least one universal coordinate transform $\widetilde{Q} : \mathbb{M}k(\mathfrak{l}) \mapsto \mathbb{M}k(\mathfrak{m})$ from $\mathfrak{l}$ to $\mathfrak{m}$ exists.*
In the case where reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ allow for a universal coordinate transform, we use the notation

$$\mathfrak{l} \underset{\mathfrak{C}}{\rightleftarrows} \mathfrak{m}.$$

In the case where the kinematic set $\mathfrak{C}$ is known in advance, we use the abbreviated notation $\mathfrak{l} \rightleftarrows \mathfrak{m}$.

(4) *An indexed family of mappings $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$ will be called a **universal coordinate transform for the kinematic set** $\mathfrak{C}$ if and only if the following holds.*
- *For arbitrary $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ the mapping $\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}$ is a universal coordinate transform from $\mathfrak{l}$ to $\mathfrak{m}$.*
- *For any $\mathfrak{l}, \mathfrak{m}, \mathfrak{p} \in \mathcal{L}k(\mathfrak{C})$ and $\mathrm{w} \in \mathbb{M}k(\mathfrak{l})$ the following equalities hold true:*

$$\widetilde{Q}_{\mathfrak{l},\mathfrak{l}}(\mathrm{w}) = \mathrm{w}; \quad \widetilde{Q}_{\mathfrak{p},\mathfrak{m}}\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w})\right) = \widetilde{Q}_{\mathfrak{p},\mathfrak{l}}(\mathrm{w}). \tag{2}$$

(5) *We say that a kinematic set $\mathfrak{C}$ **allows for a universal coordinate transform** if and only if there exists at least one universal coordinate transform $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$ for $\mathfrak{C}$.*

Remark 4. In the cases where the kinematic set $\mathfrak{C}$ is known in advance, we use the abbreviated notation $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega)$ instead of the notation $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C})$.

Assertion 1 ([24, 29]). *For an arbitrary precisely visible kinematic set $\mathfrak{C}$ the following propositions are equivalent:*

(1) *$\mathfrak{C}$ allows for a universal coordinate transform.*
(2) *For arbitrary reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ the relation $\mathfrak{l} \rightleftarrows \mathfrak{m}$ holds (that is, arbitrary two reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ allow for a universal coordinate transform).*
(3) *There exists a reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ such that, for any reference frame $\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$, the relation $\mathfrak{l} \rightleftarrows \mathfrak{m}$ holds.*

Let $\mathfrak{C}$ be any kinematic set. For an arbitrary reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$, we denote

$$\mathbb{T}\mathbf{rj}(\mathfrak{l};\, \mathfrak{C}) := \left\{\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})\right\}; \ \overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l};\, \mathfrak{C}) := \mathbb{M}k(\mathfrak{l}) \setminus \mathbb{T}\mathbf{rj}(\mathfrak{l};\, \mathfrak{C})$$

(in the cases where the kinematic set $\mathfrak{C}$ is known in advance, we use the abbreviated notations $\mathbb{T}\mathbf{rj}(\mathfrak{l})$, $\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l})$ instead of the notations $\mathbb{T}\mathbf{rj}(\mathfrak{l};\, \mathfrak{C})$, $\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l};\, \mathfrak{C})$, correspondingly). The set $\mathbb{T}\mathbf{rj}(\mathfrak{l})$ will be called a ***general trajectory*** for the reference frame $\mathfrak{l}$, and the set $\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l})$ will be called a ***complement of general trajectory*** of the reference frame $\mathfrak{l}$ in the kinematic set $\mathfrak{C}$.

Theorem 1 ([24, 29]). *Let $\mathfrak{C}$ be any precisely visible kinematic set.*

Reference frames $\mathfrak{l}, \mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ allow for a universal coordinate transform (i.e. $\mathfrak{l} \rightleftarrows \mathfrak{m}$) if and only if the following conditions are satisfied:

(1) $\mathbf{card}\left(\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{l})\right) = \mathbf{card}\left(\overline{\mathbb{T}\mathbf{rj}}(\mathfrak{m})\right)$*, where* $\mathbf{card}(\mathcal{M})$ *means the **cardinality** of the set $\mathcal{M}$.*

(2) *For arbitrary elementary-time states* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ *the equality* $\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega_1) = \mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega_2)$ *is true if and only if* $\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega_1) = \mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega_2)$.

In the next two sections we present theorems on image and multi-image, which are necessary to build mathematically strict models of kinematics of special relativity and its extension to the kinematics that allows super-light motion for inertial reference frames.

4. Theorem on image for base changeable sets

Definition 5. *An ordered triple* $(\mathbb{T}, \mathcal{X}, U)$ *will be referred to as a* ***evolution projector*** *for a base changeable set* $\mathcal{B}$ *if and only if*

1. $\mathbb{T} = (\mathbf{T}, \leq)$ *is a linearly ordered set;*
2. $\mathcal{X}$ *is any set;*
3. U *is a mapping from* $\mathbb{B}\mathfrak{s}(\mathcal{B})$ *into* $\mathbf{T} \times \mathcal{X}$ $(U : \mathbb{B}\mathfrak{s}(\mathcal{B}) \mapsto \mathbf{T} \times \mathcal{X})$.

Definition 6 ([20])**.** *Let* $\mathcal{B}$ *be any base changeable set. We will say that elementary-time states* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ *are* ***united by fate*** *in* $\mathcal{B}$ *if and only if at least one of the conditions* $\omega_2 \leftarrow \omega_1$ *or* $\omega_1 \leftarrow \omega_2$ *is satisfied.*

Theorem 2 ([23, 29])**.** *Let* $(\mathbb{T}, \mathcal{X}, U)$ *be any evolution projector for a base changeable set* $\mathcal{B}$. *Then there exists only one base changeable set* $U[\mathcal{B}, \mathbb{T}]$ *satisfying the following conditions:*

(1) $\mathbb{T}\mathbf{m}(U[\mathcal{B}, \mathbb{T}]) = \mathbb{T}$;
(2) $\mathbb{B}\mathfrak{s}(U[\mathcal{B}, \mathbb{T}]) = U(\mathbb{B}\mathfrak{s}(\mathcal{B})) = \{U(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}$;
(3) *Let* $\widetilde{\omega}_1, \widetilde{\omega}_2 \in \mathbb{B}\mathfrak{s}(U[\mathcal{B}, \mathbb{T}])$ *and* $\mathsf{tm}(\widetilde{\omega}_1) \neq \mathsf{tm}(\widetilde{\omega}_2)$. *Then* $\widetilde{\omega}_1$ *and* $\widetilde{\omega}_2$ *are united by fate in* $U[\mathcal{B}, \mathbb{T}]$ *if and only if there exist united by fate in* $\mathcal{B}$ *elementary-time states* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ *such that* $\widetilde{\omega}_1 = U(\omega_1)$, $\widetilde{\omega}_2 = U(\omega_2)$.

Remark 5. In the case where $\mathbb{T} = \mathbb{T}\mathbf{m}(\mathcal{B})$, we use the notation $U[\mathcal{B}]$ instead of the notation $U[\mathcal{B}, \mathbb{T}]$

$$U[\mathcal{B}] := U[\mathcal{B}, \mathbb{T}\mathbf{m}(\mathcal{B})].$$

Remark 6. Let $\mathcal{B}$ be any base changeable set and $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})} : \mathbb{B}\mathfrak{s}(\mathcal{B}) \mapsto \mathbf{Tm}(\mathcal{B}) \times \mathfrak{B}\mathfrak{s}(\mathcal{B})$ be a mapping given by the formula $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}(\omega) = \omega$ $(\omega \in \mathbb{B}\mathfrak{s}(\mathcal{B}))$. Then the triple

$$\left(\mathbb{T}\mathbf{m}(\mathcal{B}), \mathfrak{B}\mathfrak{s}(\mathcal{B}), \mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}\right),$$

is, apparently, an evolution projector for $\mathcal{B}$. Moreover, if we substitute $\mathbb{T}\mathbf{m}(\mathcal{B})$ and $\mathcal{B}$ into Theorem 2 instead of $\mathbb{T}$ and $U[\mathcal{B}, \mathbb{T}]$ (correspondingly), we can see that all conditions of this Theorem are satisfied. Hence for the identity mapping $\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}$ (on $\mathbb{B}\mathfrak{s}(\mathcal{B})$), we obtain

$$\mathbb{I}_{\mathbb{B}\mathfrak{s}(\mathcal{B})}[\mathcal{B}] = \mathcal{B}.$$

5. Theorem on multi-image for kinematic sets

Further $\mathfrak{R}(U)$ will mean the ***range*** of an arbitrary mapping U.

Definition 7. *The evolution projector* $(\mathbb{T}, \mathcal{X}, U)$ *(where* $\mathbb{T} = (\mathbf{T}, \leq)$*) for a base changeable set* $\mathcal{B}$ *will be called* ***injective*** *if and only if the mapping* U *is an injection from* $\mathbb{B}\mathfrak{s}(\mathcal{B})$ *to* $\mathbf{T} \times \mathcal{X}$ *(that is, a bijection from* $\mathbb{B}\mathfrak{s}(\mathcal{B})$ *onto the set* $\mathfrak{R}(U) \subseteq \mathbf{T} \times \mathcal{X}$*).*

Definition 8.

(1) *The ordered composition of five sets* $(\mathbb{T}, \mathcal{X}, U, \mathfrak{Q}, k)$ *will be called an injective* ***kinematic vector projector*** *for a base changeable set* $\mathcal{B}$ *if and only if*
  1.1: $(\mathbb{T}, \mathcal{X}, U)$ *is an injective evolution projector for* $\mathcal{B}$.
  1.2: $\mathfrak{Q} = (\mathfrak{X}, \|\cdot\|)$ *is a linear normed space.*
  1.3: k *is a mapping from* $\mathcal{X}$ *into* $\mathfrak{X}$.

(2) *Any indexed family* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ *(where* $\mathcal{A} \neq \emptyset$*) of injective kinematic vector projectors for a base changeable set* $\mathcal{B}$ *will be called a* ***kinematic vector multi-projector*** *for* $\mathcal{B}$.

Remark 7. Henceforward we will consider only injective kinematic vector projectors. That is why we will use the term "kinematic projector" instead of the term "injective kinematic vector projector". Also we will use the term "kinematic multi-projector" instead of "kinematic vector multi-projector".

Theorem 3 ([28, 29]). *Let* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ *be a kinematic multi-projector for a base changeable set* $\mathcal{B}$*. Then*

A) *Only one kinematic set* $\mathfrak{C}$ *exists satisfying the following conditions:*

(1) $\mathcal{L}k(\mathfrak{C}) = \{(\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \mid \alpha \in \mathcal{A}\}$.

(2) *For any reference frames* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta]) \in \mathcal{L}k(\mathfrak{C})$ *(*$\alpha, \beta \in \mathcal{A}$*) and any set* $A \subseteq \mathbb{B}\mathfrak{s}(\mathfrak{l}) = U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B}))$ *the following equality holds:*

$$\langle \mathfrak{m} \leftarrow \mathfrak{l}, \mathcal{Z} \rangle A = U_\beta\left(U_\alpha^{[-1]}(A)\right) = \left\{U_\beta\left(U_\alpha^{[-1]}(\omega)\right) \mid \omega \in A\right\},$$

where $U_\alpha^{[-1]}$ *is the mapping* ***inverse*** *to* U_α.

(3) *For any reference frame* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$ *(where* $\alpha \in \mathcal{A}$*) the following equalities hold true:*

$$\textbf{2.1)}\ (\mathbf{Zk}(\mathfrak{l}), \|\cdot\|_{\mathfrak{l}}) = \mathfrak{Q}_\alpha;\ \textbf{2.2)}\ \mathsf{q}_{\mathfrak{l}}(x) = k_\alpha(x) \quad (x \in \mathfrak{Bs}(\mathfrak{l})).$$

B) *A kinematic set* $\mathfrak{C}$ *satisfying the conditions 1, 2, 3 is precisely visible.*

Remark 8. Suppose that a kinematic set $\mathfrak{C}$ satisfies Condition 1 of Theorem 3. Then for any reference frame $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$, according to Remark 3 (item 4)), we have $\mathtt{ind}(\mathfrak{l}) = \alpha$, $\mathfrak{l}^{\hat{}} = U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]$, hence, $\mathbb{B}\mathfrak{s}(\mathfrak{l}) = \mathbb{B}\mathfrak{s}(\mathfrak{l}^{\hat{}}) = \mathbb{B}\mathfrak{s}(U_\alpha[\mathcal{B}, \mathbb{T}_\alpha])$. Therefore, by Theorem 2, we obtain $\mathbb{B}\mathfrak{s}(\mathfrak{l}) = U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B}))$. Thus, Condition 2 of Theorem 3 is correctly formulated.

Definition 9. *Let* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ *be a kinematic multi-projector for a base changeable set* $\mathcal{B}$*. A kinematic set* $\mathfrak{C}$ *satisfying the conditions 1, 2, 3 of Theorem 3 will be called a* ***kinematic multi-image*** *of the base changeable set* $\mathcal{B}$ *relatively the kinematic multi-projector* $\mathfrak{P}$*. This kinematic set will be denoted by* $\mathfrak{Kim}[\mathfrak{P}, \mathcal{B}]$

$$\mathfrak{Kim}[\mathfrak{P}, \mathcal{B}] := \mathfrak{C}.$$

Properties 1 ([28, 29]). *Let,* $\mathfrak{P} = ((\mathbb{T}_\alpha, \mathcal{X}_\alpha, U_\alpha, \mathfrak{Q}_\alpha, k_\alpha) \mid \alpha \in \mathcal{A})$ *be a kinematic multi-projector for* $\mathcal{B}$ *(where* $\mathbb{T}_\alpha = (\mathbf{T}_\alpha, \leq_\alpha)$, $\mathfrak{Q}_\alpha = \left(\mathfrak{X}_\alpha, \|\cdot\|_{(\alpha)}\right)$, $\alpha \in \mathcal{A}$*). Suppose that* $\mathfrak{C} = \mathfrak{Kim}[\mathfrak{P}, \mathcal{B}]$*. Then*

(1) $\mathcal{L}k(\mathfrak{C}) = \{(\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \mid \alpha \in \mathcal{A}\}$.

(2) $\mathcal{I}nd(\mathfrak{C}) = \mathcal{A}$.

(3) *For any reference frame* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$*, the following equalities hold:*

$$\mathbb{B}\mathfrak{s}(\mathfrak{l}) = U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B})) = \{U_\alpha(\omega) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\}; \quad \mathfrak{Bs}(\mathfrak{l}) = \{\mathsf{bs}(U_\alpha(\omega)) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\};$$

$$\mathbb{T}\mathbf{m}(\mathfrak{l}) = \mathbb{T}_\alpha; \quad \mathbf{Tm}(\mathfrak{l}) = \mathbf{T}_\alpha;$$

$$\mathbf{Zk}(\mathfrak{l}) = \mathfrak{X}_\alpha; \quad \mathbb{M}k(\mathfrak{l}) = \mathbf{Tm}(\mathfrak{l}) \times \mathbf{Zk}(\mathfrak{l}) = \mathbf{T}_\alpha \times \mathfrak{X}_\alpha;$$

$$\mathsf{q}_{\mathfrak{l}}(x) = k_\alpha(x) \quad (x \in \mathfrak{Bs}(\mathfrak{l}));$$

$$\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega) = (\mathsf{tm}(\omega), \mathsf{q}_{\mathfrak{l}}(\mathsf{bs}(\omega))) = (\mathsf{tm}(\omega), k_\alpha(\mathsf{bs}(\omega))) \quad (\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})).$$

(4) *Let* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$, *where* $\alpha \in \mathcal{A}$. *Suppose that* $\widetilde{\omega}_1, \widetilde{\omega}_2 \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ *and* $\mathsf{tm}(\widetilde{\omega}_1) \neq \mathsf{tm}(\widetilde{\omega}_2)$. *Then* $\widetilde{\omega}_1$ *and* $\widetilde{\omega}_2$ *are united by fate in* $\mathfrak{l}$ *if and only if there exist united by fate in* $\mathcal{B}$ *elementary-time states* $\omega_1, \omega_2 \in \mathbb{B}\mathfrak{s}(\mathcal{B})$ *such that* $\widetilde{\omega}_1 = U_\alpha(\omega_1)$, $\widetilde{\omega}_2 = U_\alpha(\omega_2)$.

(5) *For any reference frames* $\mathfrak{l} = (\alpha, U_\alpha[\mathcal{B}, \mathbb{T}_\alpha]) \in \mathcal{L}k(\mathfrak{C})$, $\mathfrak{m} = (\beta, U_\beta[\mathcal{B}, \mathbb{T}_\beta]) \in \mathcal{L}k(\mathfrak{C})$ $(\alpha, \beta \in \mathcal{A})$ *the following equality holds:*

$$\langle !\, \mathfrak{m} \leftarrow \mathfrak{l},\, \mathfrak{C} \rangle\, \omega = U_\beta\left(U_\alpha^{[-1]}(\omega)\right) \quad (\omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l}) = U_\alpha(\mathbb{B}\mathfrak{s}(\mathcal{B}))).$$

6. Kinematic sets, generated by special relativity and its tachyon extensions

Let $(\mathfrak{X}, \|\cdot\|)$ be a linear normed space and $\mathcal{B}$ be a base changeable set such that $\mathfrak{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathfrak{X}$ (such a base changeable set $\mathcal{B}$ exists, because, for example, we may put $\mathcal{B} := \mathcal{A}t(\mathbb{T}, \mathcal{R})$, where $\mathcal{R}$ is a system of abstract trajectories from some linearly ordered set $\mathbb{T}$ to a set $\mathbf{M} \subseteq \mathfrak{X}$, where the definition of $\mathcal{A}t(\mathbb{T}, \mathcal{R})$ can be found in [22,29]). Let $\mathbb{U}$ be any transforming set of bijections (in the sense of [28, formula (16)] or [29, Example I.11.2]) relatively $\mathcal{B}$ on $\mathfrak{X}$, that is, any mapping $\mathbf{U} \in \mathbb{U}$ is a bijection of the form $\mathbf{U} : \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X} \longleftrightarrow \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X}$. Then we have $\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X}$. Hence, the set of bijections $\mathbb{U}$ generates a kinematic multi-projector $\widehat{\mathbb{U}} := ((\mathbb{T}\mathbf{m}(\mathcal{B}), \mathfrak{X}, \mathbf{U}, (\mathfrak{X}, \|\cdot\|), \mathbb{I}_{\mathfrak{X}}) \mid \mathbf{U} \in \mathbb{U})$ for $\mathcal{B}$, where $\mathbb{I}_{\mathfrak{X}}$ is the identity mapping on $\mathfrak{X}$. Denote

$$\mathfrak{Kim}(\mathbb{U}, \mathcal{B}; \mathfrak{X}) := \mathfrak{Kim}\left[\widehat{\mathbb{U}}, \mathcal{B}\right]. \tag{3}$$

Theorem 4. *The kinematic set* $\mathfrak{C} = \mathfrak{Kim}(\mathbb{U}, \mathcal{B}; \mathfrak{X})$ *allows for a universal coordinate transform. Moreover,* $\mathcal{L}k(\mathfrak{C}) = ((\mathbf{U}, \mathbf{U}[\mathcal{B}]) \mid \mathbf{U} \in \mathbb{U})$, *and the system of mappings*

$$\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})}$$

defined by

$$\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}(\mathrm{w}) = \mathbf{V}\left(\mathbf{U}^{[-1]}(\mathrm{w})\right), \quad \mathrm{w} \in \mathbb{M}k(\mathfrak{l}) = \mathbf{Tm}(\mathcal{B}) \times \mathfrak{X} \tag{4}$$
$$(\, \mathfrak{l} = (\mathbf{U}, \mathbf{U}[\mathcal{B}]) \in \mathcal{L}k(\mathfrak{C}), \quad \mathfrak{m} = (\mathbf{V}, \mathbf{V}[\mathcal{B}]) \in \mathcal{L}k(\mathfrak{C})\,),$$

is a universal coordinate transform for $\mathfrak{C}$.

Proof. Let $(\mathfrak{X}, \|\cdot\|)$ be a linear normed space and $\mathbb{U}$ a transforming set of bijections relatively to a base changeable set $\mathcal{B}$ $(\mathfrak{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathfrak{X})$ on $\mathfrak{X}$. Denote $\mathfrak{C} := \mathfrak{Kim}(\mathbb{U}, \mathcal{B}; \mathfrak{X})$. Then $\mathfrak{C} = \mathfrak{Kim}\left[\widehat{\mathbb{U}}, \mathcal{B}\right]$, where $\widehat{\mathbb{U}} = ((\mathbb{T}\mathbf{m}(\mathcal{B}), \mathfrak{X}, \mathbf{U}, (\mathfrak{X}, \|\cdot\|), \mathbb{I}_{\mathfrak{X}}) \mid \mathbf{U} \in \mathbb{U})$. Hence, according to Property 1(1) [5], $\mathcal{L}k(\mathfrak{C}) = \{(\mathbf{U}, \mathbf{U}[\mathcal{B}]) \mid \mathbf{U} \in \mathbb{U}\}$. And, by Property 1(3), for an arbitrary reference frame $\mathfrak{l} = (\mathbf{U}, \mathbf{U}[\mathcal{B}]) \in \mathcal{L}k(\mathfrak{C})$ we have $\mathfrak{B}\mathfrak{s}(\mathfrak{l}) = \{\mathsf{bs}(\mathbf{U}(\omega)) \mid \omega \in \mathbb{B}\mathfrak{s}(\mathcal{B})\} \subseteq \mathfrak{X}$. Herewith, by Theorem 3, $\mathfrak{q}_\mathfrak{l}(x, \mathfrak{C}) = x$ $(\forall x \in \mathfrak{B}\mathfrak{s}(\mathfrak{l}))$. Hence

$$\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega; \mathfrak{C}) = (\mathsf{tm}(\omega), \mathfrak{q}_\mathfrak{l}(\mathsf{bs}(\omega))) = (\mathsf{tm}(\omega), \mathsf{bs}(\omega)) = \omega \quad (\mathfrak{l} \in \mathcal{L}k(\mathfrak{C}),\ \omega \in \mathbb{B}\mathfrak{s}(\mathfrak{l})).$$

Using the last equality and Property 1(5), for arbitrary reference frames $\mathfrak{l} = (\mathbf{U}, \mathbf{U}[\mathcal{B}]) \in \mathcal{L}k(\mathfrak{C})$, $\mathfrak{m} = (\mathbf{V}, \mathbf{V}[\mathcal{B}]) \in \mathcal{L}k(\mathfrak{C})$ $(\mathbf{U}, \mathbf{V} \in \mathbb{U})$ we obtain

$$\begin{aligned}\mathbf{Q}^{\langle \mathfrak{m} \leftarrow \mathfrak{l} \rangle}(\omega; \mathfrak{C}) &= \mathbf{Q}^{\langle \mathfrak{m} \rangle}(\langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega) = \langle !\, \mathfrak{m} \leftarrow \mathfrak{l} \rangle\, \omega \\ &= \mathbf{V}\left(\mathbf{U}^{[-1]}(\omega)\right) = \mathbf{V}\left(\mathbf{U}^{[-1]}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)\right)\right) = \widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\left(\mathbf{Q}^{\langle \mathfrak{l} \rangle}(\omega)\right).\end{aligned}$$

[5] For example, reference to Property 1(3) means reference to the item 3 from the group of properties "Properties 1".

It is not hard to verify that the system of mappings $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathfrak{C})}$ satisfies the conditions (2). Therefore, by Definition 4 (item 4), we see that $\left(\widetilde{Q}_{\mathfrak{m},\mathfrak{l}}\right)_{\mathfrak{l},\mathfrak{m}\in\mathcal{L}k(\mathfrak{C})}$ is a universal coordinate transform for $\mathfrak{C}$. $\square$

Let $(\mathfrak{H}, \|\cdot\|, \langle\cdot,\cdot\rangle)$ be a Hilbert space over the real number field, $\dim(\mathfrak{H}) \geq 1$, and $\mathcal{L}(\mathfrak{H})$ be the space of (homogeneous) linear continuous operators on the space $\mathfrak{H}$. Denote by $\mathcal{L}^{\times}(\mathfrak{H})$ the space of all operators of affine transformations over the space $\mathfrak{H}$, that is, $\mathcal{L}^{\times}(\mathfrak{H}) = \left\{\mathbf{A}_{[\mathbf{a}]} \mid \mathbf{A} \in \mathcal{L}(\mathfrak{H}),\, \mathbf{a} \in \mathfrak{H}\right\}$, where $\mathbf{A}_{[\mathbf{a}]}x = \mathbf{A}x + \mathbf{a}$, $x \in \mathfrak{H}$. A ***Minkowski space*** over the Hilbert space $\mathfrak{H}$ is defined as the Hilbert space $\mathcal{M}(\mathfrak{H}) = \mathbb{R} \times \mathfrak{H} = \left\{(t,x) \mid t \in \mathbb{R},\, x \in \mathfrak{H}\right\}$, equipped with the inner product and the norm: $\langle \mathrm{w}_1, \mathrm{w}_2\rangle = \langle \mathrm{w}_1, \mathrm{w}_2\rangle_{\mathcal{M}(\mathfrak{H})} = t_1 t_2 + \langle x_1, x_2\rangle$, $\|\mathrm{w}_1\| = \|\mathrm{w}_1\|_{\mathcal{M}(\mathfrak{H})} = \left(t_1^2 + \|x_1\|^2\right)^{1/2}$ (where $\mathrm{w}_i = (t_i, x_i) \in \mathcal{M}(\mathfrak{H})$, $i \in \{1,2\}$) ([17, 21]). In the space $\mathcal{M}(\mathfrak{H})$ we select the following subspaces:

$$\mathfrak{H}_0 := \{(t, \mathbf{0}) \mid t \in \mathbb{R}\}, \quad \mathfrak{H}_1 := \{(0, x) \mid x \in \mathfrak{H}\},$$

with $\mathbf{0}$ being the zero vector. Then, $\mathcal{M}(\mathfrak{H}) = \mathfrak{H}_0 \oplus \mathfrak{H}_1$, where $\oplus$ means the orthogonal sum of subspaces. Denote $\mathbf{e}_0 := (1, \mathbf{0}) \in \mathcal{M}(\mathfrak{H})$. Now, we introduce orthogonal projectors on the subspaces $\mathfrak{H}_0$ and $\mathfrak{H}_1$,

$$\mathbf{X}\mathrm{w} := (0, x) \in \mathfrak{H}_1; \quad \widehat{\mathbf{T}}\mathrm{w} := (t, \mathbf{0}) = \mathcal{T}(\mathrm{w})\,\mathbf{e}_0 \in \mathfrak{H}_0,$$
$$\text{where} \quad \mathcal{T}(\mathrm{w}) := t \quad (\mathrm{w} = (t, x) \in \mathcal{M}(\mathfrak{H})).$$

Any vector $V \in \mathfrak{H}_1$ generates the following subspaces in the space $\mathfrak{H}_1$:

$$\mathfrak{H}_1[V] = \mathbf{span}\,\{V\}; \quad \mathfrak{H}_{1\perp}[V] = \mathfrak{H}_1 \ominus \mathfrak{H}_1[V] = \{x \in \mathfrak{H}_1 \mid \langle x, V\rangle = 0\},$$

where $\mathbf{span}\,M$ denotes the linear span of the set $M \subseteq \mathcal{M}(\mathfrak{H})$. We will denote by $\mathbf{X}_1[V]$ and $\mathbf{X}_1^{\perp}[V]$ the orthogonal projectors onto the subspaces $\mathfrak{H}_1[V]$ and $\mathfrak{H}_{1\perp}[V]$

$$\mathbf{X}_1[V]\,\mathrm{w} = \begin{cases} \langle V, \mathrm{w}\rangle \|V\|^{-2} V, & V \neq \mathbf{0} \\ \mathbf{0}, & V = \mathbf{0} \end{cases}, \quad \mathrm{w} \in \mathcal{M}(\mathfrak{H}); \quad \mathbf{X}_1^{\perp}[V] = \mathbf{X} - \mathbf{X}_1[V].$$

Then for any vector $V \in \mathfrak{H}_1$ we obtain the equality

$$\widehat{\mathbf{T}} + \mathbf{X} = \widehat{\mathbf{T}} + \mathbf{X}_1[V] + \mathbf{X}_1^{\perp}[V] = \mathbb{I}_{\mathcal{M}(\mathfrak{H})},$$

where $\mathbb{I}_{\mathcal{M}(\mathfrak{H})}$ is the identity operator on $\mathcal{M}(\mathfrak{H})$.

Denote by $\mathbf{Pk}(\mathfrak{H})$ the set of all operators $\mathbf{S} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$, that have a continuous inverse $\mathbf{S}^{-1} \in \mathcal{L}^{\times}(\mathcal{M}(\mathfrak{H}))$. Operators $\mathbf{S} \in \mathbf{Pk}(\mathfrak{H})$ will be called ***coordinate transform operators***. Let $\mathcal{B}$ be any base changeable set such that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}$ and $\mathbb{Tm}(\mathcal{B}) = (\mathbb{R}, \leq)$, where $\leq$ is the standard order in the field of real numbers $\mathbb{R}$. Then $\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbb{R} \times \mathfrak{H} = \mathcal{M}(\mathfrak{H})$. Any set $\mathbb{S} \subseteq \mathbf{Pk}(\mathfrak{H})$ is a transforming set of bijections relatively to $\mathcal{B}$ on $\mathfrak{H}$. Therefore, according to (3), the kinematic set $\mathfrak{Kim}(\mathbb{S}, \mathcal{B}; \mathfrak{H})$ exists. Now, we deduce the following corollary from Theorem 4.

Corollary 1. *The kinematic set* $\mathfrak{Kim}(\mathbb{S}, \mathcal{B}; \mathfrak{H})$ *allows for a universal coordinate transform.*

Recall that an operator $U \in \mathcal{L}(\mathfrak{H})$ is called ***unitary*** on $\mathfrak{H}$ if and only if $\exists\, U^{-1} \in \mathcal{L}(\mathfrak{H})$ and $\forall\, x \in \mathfrak{H}$ $\|Ux\| = \|x\|$. Denote

$$\mathfrak{U}(\mathfrak{H}_1) = \{U \in \mathcal{L}(\mathfrak{H}_1) \mid U \text{ is unitary on } \mathfrak{H}_1\}; \quad \mathbf{B}_1(\mathfrak{H}_1) = \{x \in \mathfrak{H}_1 \mid \|x\| = 1\}.$$

Consider any fixed values $c \in (0, \infty]$, $\lambda \in [0, \infty] \setminus \{c\}$, $s \in \{-1, 1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$. For an arbitrary vector $\mathrm{w} \in \mathcal{M}(\mathfrak{H})$ we put

$$\mathbf{W}_{\lambda,c}[s, \mathbf{n}, J]\,\mathrm{w}$$

$$(5) \qquad := \begin{cases} \frac{\left(s\mathcal{T}(\mathrm{w})-\frac{\lambda}{c^2}\langle \mathbf{n},\mathrm{w}\rangle\right)}{\sqrt{\left|1-\frac{\lambda^2}{c^2}\right|}}\mathbf{e}_0 + J\left(\frac{\lambda\mathcal{T}(\mathrm{w})-s\langle \mathbf{n},\mathrm{w}\rangle}{\sqrt{\left|1-\frac{\lambda^2}{c^2}\right|}}\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w}\right), & \lambda<\infty,\ c<\infty; \\ -\frac{\langle \mathbf{n},\mathrm{w}\rangle}{c}\mathbf{e}_0 + J\left(c\mathcal{T}(\mathrm{w})\,\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w}\right), & \lambda=\infty,\ c<\infty; \\ s\mathcal{T}(\mathrm{w})\mathbf{e}_0 + J\left((\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n},\mathrm{w}\rangle)\,\mathbf{n} + \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w}\right), & \lambda<\infty,\ c=\infty. \end{cases}$$

$$(6) \qquad \mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w} := \mathbf{W}_{\lambda,c}[s,\mathbf{n},J](\mathrm{w}+\mathbf{a}).$$

If $c<\infty$, the operators of kind $\mathbf{W}_{\lambda,c}[s,\mathbf{n},J]$ are generalized Lorentz transforms introduced in [17] (or in the papers [16, 30, 39, 40, 46], for the case $1 \le \dim(\mathfrak{H}) \le 3$). Under the additional conditions $\lambda < c < \infty$, $\dim(\mathfrak{H}) = 3$, $s = 1$, formula (5) is equivalent to the formula (28b) from [36, page 43]. That is why, in this case we obtain the classical Lorentz transforms for inertial reference frame in the most general form (with arbitrary orientation of axes). Moreover, in the case $\dim(\mathfrak{H}) = 3$, $c<\infty$ the class of operators $\mathfrak{O}_+(\mathfrak{H},c) = \{\mathbf{W}_{\lambda,c}[s,\mathbf{n},J] \mid s=1,\ \lambda<c\}$ coincides with the full Lorentz group in the sense of [37] (for more details see [23, 29]). The operators of kind $\mathbf{W}_{\lambda,\infty}[s,\mathbf{n},J]$ $(\lambda<\infty)$ are Galilean transforms (it is not difficult prove that $\mathbf{W}_{\lambda,\infty}[s,\mathbf{n},J] = \lim_{c\to\infty}\mathbf{W}_{\lambda,c}[s,\mathbf{n},J]$, where the convergence is understood in the sense of uniform operator topology).

Assertion 2 ([28, 29])**.** *For any $c\in(0,\infty]$, $\lambda\in[0,\infty]\setminus\{c\}$, $s\in\{-1,1\}$, $J\in\mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1)$, and $\mathbf{a}\in\mathcal{M}(\mathfrak{H})$ it is true that*

$$\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}] \in \mathbf{Pk}(\mathfrak{H}).$$

For $0<c\le\infty$ we introduce the following classes of operators:

$$\begin{aligned} \mathfrak{PT}(\mathfrak{H},c) &:= \{\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}] \mid s\in\{-1,1\},\ \lambda\in[0,\infty]\setminus\{c\}, \\ &\qquad\qquad \mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1),\ J\in\mathfrak{U}(\mathfrak{H}_1),\ \mathbf{a}\in\mathcal{M}(\mathfrak{H})\}; \\ \mathfrak{PT}_+(\mathfrak{H},c) &:= \{\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}] \in \mathfrak{PT}(\mathfrak{H},c) \mid s=1\}; \\ \mathfrak{P}(\mathfrak{H},c) &:= \{\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}] \in \mathfrak{PT}(\mathfrak{H},c) \mid \lambda<c\}; \\ \mathfrak{P}_+(\mathfrak{H},c) &:= \{\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}] \in \mathfrak{P}(\mathfrak{H},c) \mid s=1\}. \end{aligned}$$

(It is apparent that $\mathfrak{PT}(\mathfrak{H},\infty) = \mathfrak{P}(\mathfrak{H},\infty)$, $\mathfrak{PT}_+(\mathfrak{H},\infty) = \mathfrak{P}_+(\mathfrak{H},\infty)$). Using the introduced classes of operators, we may define the following kinematic sets:

$$\begin{aligned} \mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c) &:= \mathfrak{Kim}(\mathfrak{PT}(\mathfrak{H},c),\mathcal{B};\mathfrak{H}); \\ \mathfrak{KPT}(\mathfrak{H},\mathcal{B},c) &:= \mathfrak{Kim}(\mathfrak{PT}_+(\mathfrak{H},c),\mathcal{B};\mathfrak{H}); \\ \mathfrak{KP}_0(\mathfrak{H},\mathcal{B},c) &:= \mathfrak{Kim}(\mathfrak{P}(\mathfrak{H},c),\mathcal{B};\mathfrak{H}); \\ \mathfrak{KP}(\mathfrak{H},\mathcal{B},c) &:= \mathfrak{Kim}(\mathfrak{P}_+(\mathfrak{H},c),\mathcal{B};\mathfrak{H}). \end{aligned}$$

In the case where $\dim(\mathfrak{H}) = 3$, $c<\infty$, the kinematic set $\mathfrak{KP}(\mathfrak{H},\mathcal{B},c)$ represents the simplest mathematically strict model of the kinematics of special relativity theory in inertial frames of reference. The kinematic set $\mathfrak{KP}_0(\mathfrak{H},\mathcal{B},c)$ is constructed on the basis of the general Lorentz-Poincare group, and it includes, apart from the usual reference frames (with positive direction of time) that have understandable physical interpretation, also reference frames with negative direction of time. The kinematic sets $\mathfrak{KPT}(\mathfrak{H},\mathcal{B},c)$ and $\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c)$ include, apart from standard ("tardyon") reference frames, also "tachyon" reference frames that are moving relatively to the "tardyon" reference frames with a velocity greater than the velocity of light c. The kinematic set $\mathfrak{KP}(\mathfrak{H},\mathcal{B},\infty) = \mathfrak{KPT}(\mathfrak{H},\mathcal{B},\infty)$, in the case where $\dim(\mathfrak{H}) = 3$, $c=\infty$, represents a mathematically strict model of the Galilean kinematics in the inertial frames of reference. The next corollary follows from Theorem 4.

Corollary 2. *The kinematic sets*

$$\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c), \quad \mathfrak{KPT}(\mathfrak{H},\mathcal{B},c), \quad \mathfrak{KP}_0(\mathfrak{H},\mathcal{B},c), \quad \mathfrak{KP}(\mathfrak{H},\mathcal{B},c)$$

allow for a universal coordinate transform.

Remark 9. From results of the works [23,29] it follows that the sets of operators $\mathfrak{P}(\mathfrak{H},c)$ and $\mathfrak{P}_+(\mathfrak{H},c)$ form groups of operators over the space $\mathcal{M}(\mathfrak{H})$ (in particular for $\dim(\mathfrak{H}) = 3$ the group of operators $\mathfrak{P}_+(\mathfrak{H},c)$ coincides with the classical Poincare group in the four-dimensional Minkowski space-time). At the same time, in [21,29] it is proved that classes of the operators $\mathfrak{PT}(\mathfrak{H},c)$ and $\mathfrak{PT}_+(\mathfrak{H},c)$ do not form any group over $\mathcal{M}(\mathfrak{H})$. This means that the kinematics $\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c)$ and $\mathfrak{KPT}(\mathfrak{H},\mathcal{B},c)$ constructed on the basis of these classes do not satisfy the relativity principle, because, according to Theorem 4, the subset $\mathbb{UP}(\mathfrak{l})$ of the universal coordinate transforms (4) that provide a transition from one reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{C})$ ($\mathfrak{C} \in \{\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c), \mathfrak{KPT}(\mathfrak{H},\mathcal{B},c)\}$) to all other frames $\mathfrak{m} \in \mathcal{L}k(\mathfrak{C})$ is different for different frames $\mathfrak{l}$. But, in the kinematics $\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c)$ and $\mathfrak{KPT}(\mathfrak{H},\mathcal{B},c)$, the relativity principle is violated only in the superluminal range, because the kinematics sets $\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c)$ and $\mathfrak{KPT}(\mathfrak{H},\mathcal{B},c)$ are formed by the "addition" of new superlight reference frames to the kinematics sets $\mathfrak{KP}_0(\mathfrak{H},\mathcal{B},c)$ and $\mathfrak{KP}(\mathfrak{H},\mathcal{B},c)$ that satisfy the principle of relativity. It should be noted that the principle of relativity is the only one of experimentally established facts. Therefore, it is possible that this principle is not satisfied when we exit out of the light barrier. Possibility of revision of the relativity principle is now discussed in the physical literature (see for example, [3–5, 9, 31, 32, 44]).

7. Kinematic sets that do not allow a universal coordinate transform

In this section, we construct one interesting class of kinematic sets, in which every particle at each time moment can have its own "velocity of light". On a physical level, similar models (with particle-dependent velocity of light) were considered in the papers [10, 11, 14, 15, 45].

Let a set $\mathfrak{V}_\mathfrak{f} \subseteq (0,\infty]$ be such that

$$\mathfrak{V}_\mathfrak{f} \neq \emptyset \quad \text{and} \quad (0,\infty] \setminus \mathfrak{V}_\mathfrak{f} \neq \emptyset.$$

Denote

$$\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}} := \mathfrak{H} \times \mathfrak{V}_\mathfrak{f} = \{(x,c) \mid x \in \mathfrak{H},\ c \in \mathfrak{V}_\mathfrak{f}\}; \quad \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right) := \mathbb{R} \times \mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}.$$

The set $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ will be called the Minkowski space ***with the set of forbidden velocities*** $\mathfrak{V}_\mathfrak{f}$ over $\mathfrak{H}$. The set $\widetilde{\mathfrak{V}_\mathfrak{f}} := [0,\infty] \setminus \mathfrak{V}_\mathfrak{f}$ will be called a ***set of allowed velocities*** for the space $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. The set $\widetilde{\mathfrak{V}_\mathfrak{f}}$ is always nonempty, because, according to definition of $\widetilde{\mathfrak{V}_\mathfrak{f}}$, we have guaranteed $0 \in \widetilde{\mathfrak{V}_\mathfrak{f}}$. So, we introduce a ***set of finite nonzero allowed velocities*** for the space $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$

$$\widetilde{\mathfrak{V}_\mathfrak{f}}^{\mathsf{fin}+} := \widetilde{\mathfrak{V}_\mathfrak{f}} \setminus \{0,\infty\} = (0,\infty) \setminus \mathfrak{V}_\mathfrak{f}.$$

For an arbitrary $\omega = (t,(x,c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ we put $\omega^* := (t,x) \in \mathcal{M}(\mathfrak{H})$. Also for $\lambda \in \widetilde{\mathfrak{V}_\mathfrak{f}}$, $s \in \{-1,1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$, and $\omega = (t,(x,c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$, we introduce the notation

$$\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]\,\omega := \left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\,\omega^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\,\omega^*\right),c\right)\right). \tag{7}$$

Therefore, for any $\omega = (t,(x,c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ we have the identity

$$\left(\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]\,\omega\right)^* = \mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\,\omega^*. \tag{8}$$

Assertion 3. *For arbitrary* $\lambda \in \widetilde{\mathfrak{V}_\mathfrak{f}}$, $s \in \{-1,1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$, $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$, *the mapping* $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]$ *is a bijection on* $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$.

Proof. Suppose that $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]\,\omega_1 = \mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]\,\omega_2$, where

$$\omega_1 = (t_1,(x_1,c_1)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right), \quad \omega_2 = (t_2,(x_2,c_2)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right).$$

Then,

$$\begin{aligned}&\left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_1^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_1^*\right),c_1\right)\right)\\ &\quad = \left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\omega_2^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\omega_2^*\right),c_2\right)\right).\end{aligned}$$

Consequently, $c_1 = c_2$. Hence, we have proved the equalities

$$\begin{aligned}\mathsf{tm}\left(\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_1^*\right) &= \mathsf{tm}\left(\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_2^*\right),\\ \mathsf{bs}\left(\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_1^*\right) &= \mathsf{bs}\left(\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_2^*\right).\end{aligned}$$

Therefore, $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_1^* = \mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\omega_2^*$. And, taking into account the fact that the mapping $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]$ is a bijection on $\mathcal{M}(\mathfrak{H})$, we conclude that, $\omega_1^* = \omega_2^*$, i.e., $t_1 = t_2$, $x_1 = x_2$. Hence, $\omega_1 = (t_1,(x_1,c_1)) = (t_2,(x_2,c_2)) = \omega_2$. Thus, the mapping $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]$ is a one-to-one correspondence.

Now it remains to prove that $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]$ reflects the set $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ on $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. Consider any $\omega = (t,(x,c)) \in \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. Denote

$$\widetilde{\omega} := \left(\mathsf{tm}\left(\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\right)^{[-1]}\omega^*\right),\ \left(\mathsf{bs}\left(\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\right)^{[-1]}\omega^*\right),c\right)\right).$$

Then

$$\begin{aligned}\widetilde{\omega}^* &= \left(\mathsf{tm}\left(\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\right)^{[-1]}\omega^*\right),\ \mathsf{bs}\left(\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\right)^{[-1]}\omega^*\right)\right)\\ &= \left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\right)^{[-1]}\omega^*.\end{aligned}$$

Consequently, $\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\,\widetilde{\omega}^* = \omega^*$. Hence,

$$\begin{aligned}\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]\,\widetilde{\omega} &= \left(\mathsf{tm}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\,\widetilde{\omega}^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{\lambda,c}[s,\mathbf{n},J;\mathbf{a}]\,\widetilde{\omega}^*\right),c\right)\right)\\ &= \left(\mathsf{tm}\left(\omega^*\right),\ \left(\mathsf{bs}\left(\omega^*\right),c\right)\right) = (t,(x,c)) = \omega.\end{aligned}$$

Thus $\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}]$ is a bijection from $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$ onto $\mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. □

Denote

$$\begin{aligned}\mathfrak{PT}(\mathfrak{H};\mathfrak{V}_\mathfrak{f}) &:= \Big\{\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}] \mid \lambda \in \widetilde{\mathfrak{V}_\mathfrak{f}},\ s \in \{-1,1\},\\ &\qquad\qquad J \in \mathfrak{U}(\mathfrak{H}_1),\ \mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1),\ \mathbf{a} \in \mathcal{M}(\mathfrak{H})\Big\};\\ \mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f}) &:= \left\{\mathbf{W}_{\lambda;\mathfrak{V}_\mathfrak{f}}[s,\mathbf{n},J;\mathbf{a}] \in \mathfrak{PT}(\mathfrak{H};\mathfrak{V}_\mathfrak{f}) \mid s = 1\right\}.\end{aligned}$$

Let $\mathcal{B}$ be a base changeable set such that $\mathfrak{Bs}(\mathcal{B}) \subseteq \mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}$, $\mathbb{T}\mathbf{m}(\mathcal{B}) = (\mathbb{R},\leq)$. Then we have, $\mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathbb{R}\times\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}} = \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)$. Hence, we obtain the following kinematic multi-projectors for $\mathcal{B}$:

$$\begin{aligned}&\mathfrak{PT}(\mathfrak{H};\mathfrak{V}_\mathfrak{f})^\wedge = \left(\left((\mathbb{R},\leq),\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}},\mathbf{S},(\mathfrak{H},\|\cdot\|),\mathbf{q}\right) \mid \mathbf{S} \in \mathfrak{PT}(\mathfrak{H};\mathfrak{V}_\mathfrak{f})\right);\\ (9)\qquad &\mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})^\wedge = \left(\left((\mathbb{R},\leq),\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}},\mathbf{S},(\mathfrak{H},\|\cdot\|),\mathbf{q}\right) \mid \mathbf{S} \in \mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})\right), \quad \text{where}\\ &\qquad \mathbf{q}(\widetilde{x}) = x \quad \left(\forall\, \widetilde{x} = (x,c) \in \mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right).\end{aligned}$$

In accordance with Theorem 3, we can denote

$$\begin{aligned}\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}) &:= \mathfrak{Kim}\left[\mathfrak{PT}(\mathfrak{H};\mathfrak{V}_\mathfrak{f})^\wedge,\mathcal{B}\right];\\ \mathfrak{KPT}(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}) &:= \mathfrak{Kim}\left[\mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})^\wedge,\mathcal{B}\right].\end{aligned}$$

It turns out that the kinematic sets $\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})$ and $\mathfrak{KPT}(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})$, in the general case, do not allow for a universal coordinate transform. More precisely, they allow for a universal coordinate transform if and only if only one value of the forbidden velocity

$c \in (0,\infty]$ is actually realized. In the latter case, the kinematics in $\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})$ or $\mathfrak{KPT}(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})$ can be reduced to a kinematics of type $\mathfrak{KPT}_0(\mathfrak{H},\mathcal{B},c)$ or $\mathfrak{KPT}(\mathfrak{H},\mathcal{B},c)$.

Theorem 5. *Let the set of forbidden velocities $\mathfrak{V}_\mathfrak{f}$ be such that the corresponding set $\widetilde{\mathfrak{V}_\mathfrak{f}}^{\mathsf{fin}+} = (0,\infty)\setminus\mathfrak{V}_\mathfrak{f}$ (of finite nonzero allowed velocities) contains at least two elements (that is,* $\mathbf{card}\left(\widetilde{\mathfrak{V}_\mathfrak{f}}^{\mathsf{fin}+}\right)\geq 2$*).*

The kinematic set $\mathfrak{KPT}(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})$ allows for a universal coordinate transform if and only if there don't exist elementary states $\widetilde{x}_1=(x_1,c_1)$, $\widetilde{x}_2=(x_2,c_2)\in\mathfrak{Bs}(\mathcal{B})$ such that $c_1\neq c_2$.

To prove Theorem 5 we need two auxiliary lemmas. To formulate these lemmas we introduce the following notation.

Notation. For $y_1,y_2\in(0,\infty]$ such that $y_1\neq\infty$ or $y_2\neq\infty$, we put

$$\sigma(y_1,y_2)=\begin{cases}\left(\frac{y_1^{-2}+y_2^{-2}}{2}\right)^{-\frac12}, & y_1,y_2<\infty,\\ \sqrt{2}\,y_1, & y_1<\infty,\ y_2=\infty,\\ \sqrt{2}\,y_2, & y_1=\infty,\ y_2<\infty.\end{cases}$$

Lemma 1. *Chose any fixed $c_1,c_2\in(0,\infty]$, $c_1\neq c_2$; $\lambda\in(0,\infty)\setminus\{c_1,c_2,\sigma(c_1,c_2)\}$; $s\in\{-1,1\}$ and $J\in\mathfrak{U}(\mathfrak{H}_1)$.*

Then, for arbitrary vectors $\mathrm{w}_1,\mathrm{w}_2\in\mathcal{M}(\mathfrak{H})$ such that $\mathrm{w}_1\neq\mathrm{w}_2$ there exist $\mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a}\in\mathcal{M}(\mathfrak{H})$ for which the following equality holds:

$$\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}_1=\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}_2.$$

Proof. Further, for convenience, we assume that $c_1<c_2$. Obviously, this assumption does not restrict the generality of our conclusions.

1. At first, we are going to prove Lemma in the special case $\mathrm{w}_1=\mathbf{0}$, $\mathrm{w}_2=\mathrm{w}\neq\mathbf{0}$. Consider any fixed $\lambda\in(0,\infty)\setminus\{c_1,c_2,\sigma(c_1,c_2)\}$. According to the specifics of this case, we should find $\mathbf{n}\in\mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a}\in\mathcal{M}(\mathfrak{H})$ such that

$$\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathbf{0}=\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}.$$

Taking into account (6), we can rewrite the last condition in the form

$$\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathbf{a}=\mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,(\mathrm{w}+\mathbf{a})\,. \tag{10}$$

Denote

$$t:=\mathcal{T}(\mathrm{w})\,,\quad x:=\mathbf{X}\mathrm{w}. \tag{11}$$

Then we can write $\mathrm{w}=t\mathbf{e}_0+x$.

Consider any fixed vector $\mathbf{n}_0\in\mathbf{B}_1(\mathfrak{H}_1)$. Denote: $\mathbf{n}:=\begin{cases}\frac{x}{\|x\|}, & x\neq\mathbf{0}\\ \mathbf{n}_0, & x=\mathbf{0}\end{cases}$. Then, we have

$$\begin{aligned} x &= \|x\|\,\mathbf{n},\\ \langle\mathbf{n},\mathrm{w}\rangle &= \langle\mathbf{n},x\rangle=\|x\|\,,\\ \mathbf{X}_1^{\perp}[\mathbf{n}]\,\mathrm{w} &= \mathbf{X}\mathrm{w}-\langle\mathbf{n},\mathrm{w}\rangle\,\mathbf{n}=x-\|x\|\,\mathbf{n}=x-x=\mathbf{0}.\end{aligned} \tag{12}$$

The vector $\mathbf{a}$ is sought in the form

$$\mathbf{a}=\tau\mathbf{e}_0+\mu\mathbf{n},\quad\text{where}\quad\tau,\mu\in\mathbb{R}. \tag{13}$$

1.a) At first we consider the case $c_1,c_2<\infty$.

Substituting the value of the vector $\mathbf{a}$ from (13) into the condition (10) and applying (11), (12), (5), we obtain the following condition:

$$\begin{aligned}
&\left(s\tau-\frac{\lambda}{c_1^2}\mu\right)\gamma\left(\frac{\lambda}{c_1}\right)\mathbf{e}_0+(\lambda\tau-s\mu)\,\gamma\left(\frac{\lambda}{c_1}\right)J\mathbf{n}\\
&=\left(s\,(t+\tau)-\frac{\lambda}{c_2^2}\left(\|x\|+\mu\right)\right)\gamma\left(\frac{\lambda}{c_2}\right)\mathbf{e}_0+\left(\lambda\,(t+\tau)-s\left(\|x\|+\mu\right)\right)\gamma\left(\frac{\lambda}{c_2}\right)J\mathbf{n},
\end{aligned}\tag{14}$$

$$\text{where}\quad \gamma(\xi)=\frac{1}{\sqrt{|1-\xi^2|}},\quad \xi\ge 0,\quad \xi\ne 1.$$

Taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the following system of equations:

$$\begin{cases}\left(s\tau-\frac{\lambda}{c_1^2}\mu\right)\gamma\left(\frac{\lambda}{c_1}\right)=\left(s\,(t+\tau)-\frac{\lambda}{c_2^2}\left(\|x\|+\mu\right)\right)\gamma\left(\frac{\lambda}{c_2}\right),\\ (\lambda\tau-s\mu)\,\gamma\left(\frac{\lambda}{c_1}\right)=\left(\lambda\,(t+\tau)-s\left(\|x\|+\mu\right)\right)\gamma\left(\frac{\lambda}{c_2}\right).\end{cases}\tag{15}$$

By means of simple transformations, the system (15) can be reduced to the following equivalent form:

$$\begin{cases}\left(\gamma\left(\frac{\lambda}{c_2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)\right)\tau+\lambda s\left(\frac{\gamma\left(\frac{\lambda}{c_1}\right)}{c_1^2}-\frac{\gamma\left(\frac{\lambda}{c_2}\right)}{c_2^2}\right)\mu=\left(\frac{\lambda s\|x\|}{c_2^2}-t\right)\gamma\left(\frac{\lambda}{c_2}\right),\\ \lambda\left(\gamma\left(\frac{\lambda}{c_2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)\right)\tau+s\left(\gamma\left(\frac{\lambda}{c_1}\right)-\gamma\left(\frac{\lambda}{c_2}\right)\right)\mu=\left(s\,\|x\|-\lambda t\right)\gamma\left(\frac{\lambda}{c_2}\right).\end{cases}\tag{16}$$

The system (16) is a system of linear equations with respect to the variables "τ" and "μ". The determinant of this system may be represented as follows:

$$\begin{aligned}
\Delta=\;&-s\left(\gamma\left(\tfrac{\lambda}{c_2}\right)-\gamma\left(\tfrac{\lambda}{c_1}\right)\right)^2-s\lambda^2\left(\gamma\left(\tfrac{\lambda}{c_2}\right)-\gamma\left(\tfrac{\lambda}{c_1}\right)\right)\left(\frac{\gamma\left(\frac{\lambda}{c_1}\right)}{c_1^2}-\frac{\gamma\left(\frac{\lambda}{c_2}\right)}{c_2^2}\right)\\
=\;&-s\left(\gamma\left(\tfrac{\lambda}{c_2}\right)-\gamma\left(\tfrac{\lambda}{c_1}\right)\right)\left(\gamma\left(\tfrac{\lambda}{c_2}\right)-\gamma\left(\tfrac{\lambda}{c_1}\right)+\lambda^2\left(\frac{\gamma\left(\frac{\lambda}{c_1}\right)}{c_1^2}-\frac{\gamma\left(\frac{\lambda}{c_2}\right)}{c_2^2}\right)\right)\\
=\;&-s\left(\gamma\left(\frac{\lambda}{c_2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)\right)\left(\gamma\left(\frac{\lambda}{c_2}\right)\left(1-\frac{\lambda^2}{c_2^2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)\left(1-\frac{\lambda^2}{c_1^2}\right)\right)\\
=\;&-s\left(\gamma\left(\frac{\lambda}{c_2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)\right)\left(\frac{1-\frac{\lambda^2}{c_2^2}}{\sqrt{\left|1-\frac{\lambda^2}{c_2^2}\right|}}-\frac{1-\frac{\lambda^2}{c_1^2}}{\sqrt{\left|1-\frac{\lambda^2}{c_1^2}\right|}}\right)\\
=\;&-s\left(\gamma\left(\frac{\lambda}{c_2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)\right)\left(\Phi\left(1-\frac{\lambda^2}{c_2^2}\right)-\Phi\left(1-\frac{\lambda^2}{c_1^2}\right)\right),
\end{aligned}\tag{17}$$

where

$$\Phi(y):=\operatorname{sign}(y)\sqrt{|y|}\quad (y\in\mathbb{R}).\tag{18}$$

Since $\lambda\in(0,\infty)\setminus\{c_1,c_2,\sigma(c_1,c_2)\}$, we have that $\lambda>0$ and $\lambda\ne\sigma(c_1,c_2)$. That is why, the multiplier $\gamma\left(\frac{\lambda}{c_2}\right)-\gamma\left(\frac{\lambda}{c_1}\right)$ in (17) is nonzero. The second multiplier

$$\left(\Phi\left(1-\frac{\lambda^2}{c_2^2}\right)-\Phi\left(1-\frac{\lambda^2}{c_1^2}\right)\right)$$

also is nonzero, because $c_1<c_2$ and the function Φ is strictly monotone on $\mathbb{R}$. So, the determinant Δ of system (16) is nonzero. Therefore, this system has a unique solution. Let $(\tilde{\tau},\tilde{\mu})$ be a solution of system (16). Then, according to (13), the sought vector $\mathbf{a}$ has the form

$$\mathbf{a}=\tilde{\tau}\mathbf{e}_0+\tilde{\mu}\mathbf{n}.$$

And, substituting the obtained values of the parameters $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ into (10), we obtain a valid equality. In the case $c_2, c_2 < \infty$ and $\mathrm{w}_1 = \mathbf{0}$, Lemma is proved.

1.b) It thus remains to consider only the case $c_2 = \infty$, $c_1 < \infty$ ($\mathrm{w}_1 = \mathbf{0}$, $\mathrm{w}_2 = \mathrm{w} \neq \mathbf{0}$). Note that the case $c_1 = \infty$ is impossible, because $c_1 < c_2$.

Substituting the value of the vector $\mathbf{a}$ from (13) into the condition (10) and applying (11), (12), (5), we obtain the following condition:

$$\begin{aligned}\left(s\tau - \frac{\lambda}{c_1^2}\mu\right)\gamma\left(\frac{\lambda}{c_1}\right)\mathbf{e}_0 + (\lambda\tau - s\mu)\,\gamma\left(\frac{\lambda}{c_1}\right)\, J\mathbf{n}& \\ = s\,(t+\tau)\,\mathbf{e}_0 + \left(\lambda\,(t+\tau) - s\,(\|x\| + \mu)\right)\, J\mathbf{n}&.\end{aligned}$$

Hence, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the following system of equations:

$$\begin{cases}\left(s\tau - \frac{\lambda}{c_1^2}\mu\right)\gamma\left(\frac{\lambda}{c_1}\right) = s\,(t+\tau)\,, \\ (\lambda\tau - s\mu)\,\gamma\left(\frac{\lambda}{c_1}\right) = \lambda\,(t+\tau) - s\,(\|x\| + \mu)\,. \end{cases} \tag{19}$$

After simple transformations, system (19) can be reduced to the following equivalent form:

$$\begin{cases}\left(1 - \gamma\left(\frac{\lambda}{c_1}\right)\right)\tau + \lambda s\,\frac{\gamma\left(\frac{\lambda}{c_1}\right)}{c_1^2}\,\mu = -t, \\ \lambda\left(1 - \gamma\left(\frac{\lambda}{c_1}\right)\right)\tau + s\left(\gamma\left(\frac{\lambda}{c_1}\right) - 1\right)\mu = s\,\|x\| - \lambda t. \end{cases} \tag{20}$$

The system (20) is a system of linear equations in the variables "τ" and "μ". The determinant of this system can be represented as follows:

$$\begin{aligned}\Delta = &\; -s\left(1 - \gamma\left(\tfrac{\lambda}{c_1}\right)\right)^2 - s\lambda^2\left(1 - \gamma\left(\tfrac{\lambda}{c_1}\right)\right)\frac{\gamma\left(\frac{\lambda}{c_1}\right)}{c_1^2} \\ = &\; -s\left(1 - \gamma\left(\tfrac{\lambda}{c_1}\right)\right)\left(1 - \gamma\left(\tfrac{\lambda}{c_1}\right) + \lambda^2\,\frac{\gamma\left(\frac{\lambda}{c_1}\right)}{c_1^2}\right) \\ = &\; -s\left(1 - \gamma\left(\frac{\lambda}{c_1}\right)\right)\left(1 - \frac{1 - \frac{\lambda^2}{c_1^2}}{\sqrt{\left|1 - \frac{\lambda^2}{c_1^2}\right|}}\right) \\ = &\; -s\left(1 - \gamma\left(\frac{\lambda}{c_1}\right)\right)\left(1 - \Phi\left(1 - \frac{\lambda^2}{c_1^2}\right)\right),\end{aligned} \tag{21}$$

where the function Φ is defined by the formula (18). Since $\lambda \in (0,\infty) \setminus \{c_1, c_2, \sigma(c_1, c_2)\}$, where $c_2 = +\infty$, we see that $\lambda > 0$ and $\lambda \neq \sigma(c_1, \infty) = \sqrt{2}c_1$. That is why the multiplier $1 - \gamma\left(\frac{\lambda}{c_1}\right)$ in (21) is nonzero. The second multiplier $\left(1 - \Phi\left(1 - \frac{\lambda^2}{c_1^2}\right)\right)$ is also nonzero, because $1 = \Phi(1)$ and the function Φ is strictly monotone on $\mathbb{R}$. So, the determinant Δ of system (20) is nonzero. Therefore, this system has a unique solution. Let $(\tilde{\tau}, \tilde{\mu})$ be a solution of system (20). Then, according to (13), the needed vector $\mathbf{a}$ has the form $\mathbf{a} = \tilde{\tau}\mathbf{e}_0 + \tilde{\mu}\mathbf{n}$. And, substituting the obtained values of the parameters $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ into (10), we see that the equality holds. Hence, in the case $c_1 < \infty$, $c_2 = \infty$ and $\mathrm{w}_1 = \mathbf{0}$, Lemma also is proved.

2. We now turn to the general case where $\mathrm{w}_1, \mathrm{w}_2$ are arbitrary vectors of the space $\mathcal{M}(\mathfrak{H})$ such that $\mathrm{w}_1 \neq \mathrm{w}_2$. Let $\lambda \in (0,\infty) \setminus \{c_1, c_2, \sigma(c_1, c_2)\}$. According to the result proved in the first item of the proof of Lemma, there are vectors $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ and $\widetilde{\mathbf{a}} \in \mathcal{M}(\mathfrak{H})$ such that $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\widetilde{\mathbf{a}} = \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,(\mathrm{w}_2 - \mathrm{w}_1 + \widetilde{\mathbf{a}})$. Denote $\mathbf{a} := \widetilde{\mathbf{a}} - \mathrm{w}_1$. Then, taking into account (6), we obtain the desired equality $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}_1 = \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}_2$. □

Lemma 2. *Suppose that for some vector* $\mathrm{w} \in \mathcal{M}(\mathfrak{H})$, *we have*

$$\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} = \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w},$$

where $c_1, c_2 \in (0,\infty]$, $\lambda \in (0,\infty] \setminus \{c_1, c_2, \sigma(c_1,c_2)\}$, $s \in \{-1,1\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ *with* $c_1 \neq c_2$. *Then,* $\mathcal{T}(\mathrm{w}) = \langle \mathbf{n}, \mathrm{w} \rangle = 0$.

Proof of Lemma is subdivided into several cases. **Case 1**: $c_1, c_2 < \infty$, $\lambda < \infty$. In this case, by formula (5), we get

$$\begin{aligned}
&\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w} \\
&= \left(\left(\gamma\left(\frac{\lambda}{c_1}\right) - \gamma\left(\frac{\lambda}{c_2}\right) \right) s\mathcal{T}(\mathrm{w}) - \left(\frac{\lambda}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right) - \frac{\lambda}{c_2^2}\gamma\left(\frac{\lambda}{c_2}\right) \right) \langle \mathbf{n}, \mathrm{w} \rangle \right) \mathbf{e}_0 \\
&\quad + \left(\gamma\left(\frac{\lambda}{c_1}\right) - \gamma\left(\frac{\lambda}{c_2}\right) \right) \left(\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n}, \mathrm{w} \rangle \right) J\mathbf{n},
\end{aligned} \tag{22}$$

where the function γ is determined by formula (14). It follows from conditions of Lemma that $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w} = \mathbf{0}$, where $\mathbf{0}$ is the zero vector in the space $\mathcal{M}(\mathfrak{H})$. Hence, the right-hand side of the equality (22) is equal to the zero vector. Therefore, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the following equalities:

$$\begin{aligned}
s\left(\gamma\left(\tfrac{\lambda}{c_1}\right) - \gamma\left(\tfrac{\lambda}{c_2}\right) \right) \mathcal{T}(\mathrm{w}) - \left(\tfrac{\lambda}{c_1^2}\gamma\left(\tfrac{\lambda}{c_1}\right) - \tfrac{\lambda}{c_2^2}\gamma\left(\tfrac{\lambda}{c_2}\right) \right) \langle \mathbf{n}, \mathrm{w} \rangle &= 0, \\
\left(\gamma\left(\tfrac{\lambda}{c_1}\right) - \gamma\left(\tfrac{\lambda}{c_2}\right) \right) \ \left(\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n}, \mathrm{w} \rangle \right) &= 0.
\end{aligned} \tag{23}$$

According to the conditions of Lemma, $\lambda > 0$ and $\lambda \neq \sigma(c_1,c_2) = \sqrt{\frac{2}{\frac{1}{c_1^2} + \frac{1}{c_2^2}}}$. Consequently, $\gamma\left(\frac{\lambda}{c_1}\right) - \gamma\left(\frac{\lambda}{c_2}\right) \neq 0$. Thus, the equalities (23) can be rewritten as

$$\begin{cases}
s\left(\gamma\left(\tfrac{\lambda}{c_1}\right) - \gamma\left(\tfrac{\lambda}{c_2}\right) \right) \mathcal{T}(\mathrm{w}) - \left(\tfrac{\lambda}{c_1^2}\gamma\left(\tfrac{\lambda}{c_1}\right) - \tfrac{\lambda}{c_2^2}\gamma\left(\tfrac{\lambda}{c_2}\right) \right) \langle \mathbf{n}, \mathrm{w} \rangle = 0, \\
\lambda\mathcal{T}(\mathrm{w}) - s\langle \mathbf{n}, \mathrm{w} \rangle = 0.
\end{cases} \tag{24}$$

System (24) is a system of linear homogeneous equations in the variables $\mathcal{T}(\mathrm{w})$ and $\langle \mathbf{n}, \mathrm{w} \rangle$. The determinant of this system is

$$\begin{aligned}
\Delta &= -\left[\left(\gamma\left(\frac{\lambda}{c_1}\right) - \gamma\left(\frac{\lambda}{c_2}\right) \right) - \left(\frac{\lambda^2}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right) - \frac{\lambda^2}{c_2^2}\gamma\left(\frac{\lambda}{c_2}\right) \right) \right] \\
&= \Phi\left(1 - \frac{\lambda^2}{c_2^2}\right) - \Phi\left(1 - \frac{\lambda^2}{c_1^2}\right),
\end{aligned}$$

where the function Φ is determined by formula (18). Since the function Φ is strictly monotone on $\mathbb{R}$, the determinant Δ of system (24) is nonzero. Hence, $\mathcal{T}(\mathrm{w}) = \langle \mathbf{n}, \mathrm{w} \rangle = 0$, which was to be proved.

Case 2: $c_1, c_2 < \infty$, $\lambda = \infty$.

In this case, by formula (5), we get

$$\begin{aligned}
0 &= \mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w} \\
&= -\frac{\langle \mathbf{n}, \mathrm{w} \rangle}{c_1}\mathbf{e}_0 + c_1\mathcal{T}(\mathrm{w})\,J\mathbf{n} - \left(-\frac{\langle \mathbf{n}, \mathrm{w} \rangle}{c_2}\mathbf{e}_0 + c_2\mathcal{T}(\mathrm{w})\,J\mathbf{n} \right) \\
&= -\left(\frac{1}{c_1} - \frac{1}{c_2} \right) \langle \mathbf{n}, \mathrm{w} \rangle\,\mathbf{e}_0 + (c_1 - c_2)\,\mathcal{T}(\mathrm{w})\,J\mathbf{n}.
\end{aligned}$$

And since $c_1 \neq c_2$, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, we get the equality $\mathcal{T}(\mathrm{w}) = \langle \mathbf{n}, \mathrm{w} \rangle = 0$.

Case 3: $c_1 < \infty$, $c_2 = \infty$.

By the conditions of Lemma, $\lambda \neq c_2$. Hence, in this case, we have $\lambda < \infty$. And, according to (5), we obtain

$$
\begin{aligned}
0 &= \mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J]\,\mathrm{w} - \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J]\,\mathrm{w}\\
&= \left(\left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right)s\mathcal{T}(\mathrm{w}) - \frac{\lambda}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\langle\mathbf{n},\mathrm{w}\rangle\right)\mathbf{e}_0\\
&\quad+\left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right)(\lambda\mathcal{T}(\mathrm{w}) - s\langle\mathbf{n},\mathrm{w}\rangle)\,J\mathbf{n}.
\end{aligned}
\tag{25}
$$

By the conditions of Lemma, $\lambda > 0$ and $\lambda \neq \sigma(c_1,c_2) = \sqrt{2}\,c_1$. Thus, $\gamma\left(\frac{\lambda}{c_1}\right) - 1 \neq 0$. Hence, taking into account orthogonality of the vector $\mathbf{e}_0$ to the subspace $\mathfrak{H}_1$ and unitarity of the operator J on the subspace $\mathfrak{H}_1$, from the equality (25) we get a system of equations,

$$
\begin{cases}
\left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right)s\mathcal{T}(\mathrm{w}) - \frac{\lambda}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\langle\mathbf{n},\mathrm{w}\rangle &= 0,\\
\lambda\mathcal{T}(\mathrm{w}) - s\langle\mathbf{n},\mathrm{w}\rangle &= 0.
\end{cases}
\tag{26}
$$

System (26) is a system of linear homogeneous equations in the variables $\mathcal{T}(\mathrm{w})$ and $\langle\mathbf{n},\mathrm{w}\rangle$. The determinant of this system is

$$
\Delta_1 = -\left(\left(\gamma\left(\frac{\lambda}{c_1}\right)-1\right) - \frac{\lambda^2}{c_1^2}\gamma\left(\frac{\lambda}{c_1}\right)\right) = \Phi(1) - \Phi\left(1-\frac{\lambda^2}{c_1^2}\right).
$$

Since, by the conditions of Lemma, $\lambda > 0$ and $c_1 < \infty$, we have that $\frac{\lambda}{c_1} \neq 0$. That is why, $\Delta_1 \neq 0$. Thus, $\mathcal{T}(\mathrm{w}) = \langle\mathbf{n},\mathrm{w}\rangle = 0$.

Case 4: $c_1 = \infty$, $c_2 < \infty$ is considered similarly to the case 3.

Case $c_1, c_2 = \infty$ is impossible, because, by the conditions of Lemma, $c_1 \neq c_2$. $\square$

Corollary 3. *Let* $c_1, c_2 \in (0,\infty]$, $c_1 \neq c_2$, $s \in \{-1,1\}$, $\lambda \in (0,\infty)\setminus\{c_1,c_2,\sigma(c_1,c_2)\}$, $J \in \mathfrak{U}(\mathfrak{H}_1)$, $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$.

Then for any $\mathrm{w} \in \mathcal{M}(\mathfrak{H})$ *there exists* $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ *such that*

$$
\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w} \neq \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}.
$$

Proof. Let us consider any $\mathbf{a} \in \mathcal{M}(\mathfrak{H})$ such that

$$
\mathcal{T}(\mathrm{w}+\mathbf{a}) \neq 0. \tag{27}
$$

If we assume that $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w} = \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}$, then, according to (6), we will obtain

$$
\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J](\mathrm{w}+\mathbf{a}) = \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J](\mathrm{w}+\mathbf{a}).
$$

Hence, by Lemma 2, $\mathcal{T}(\mathrm{w}+\mathbf{a}) = 0$, contrary to (27). Thus, $\mathbf{W}_{\lambda,c_1}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w} \neq \mathbf{W}_{\lambda,c_2}[s,\mathbf{n},J;\mathbf{a}]\,\mathrm{w}$. $\square$

Proof of Theorem 5. **1**. For any fixed vector $\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1)$ we are going to prove the equality

$$
\mathbf{W}_{0;\mathfrak{V}_{\mathfrak{f}}}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}],\mathbf{0}] = \mathbb{I}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_{\mathfrak{f}}})}, \tag{28}
$$

where $\mathbb{I}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_{\mathfrak{f}}})}$ is the the identity operator on $\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_{\mathfrak{f}}})$, and

$$
\mathbb{I}_{\kappa,\mu}[\mathbf{n}]\,x := \kappa\mathbf{X}_1[\mathbf{n}]\,x + \mu\mathbf{X}_1^{\perp}[\mathbf{n}]\,x, \quad x \in \mathfrak{H}_1 \quad (\mathbf{n} \in \mathbf{B}_1(\mathfrak{H}_1),\ \kappa,\mu \in \{-1,1\}).
$$

Indeed, according to (7), (6), (5), for an arbitrary element $\omega = (t,(x,c)) \in \mathcal{M}(\mathfrak{H}_{\mathfrak{V}_{\mathfrak{f}}})$, we have

$$
\begin{aligned}
&\mathbf{W}_{0;\mathfrak{V}_{\mathfrak{f}}}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}],\mathbf{0}]\,\omega\\
&\qquad= \left(\mathsf{tm}\left(\mathbf{W}_{0,c}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}]]\,\omega^*\right),\ \left(\mathsf{bs}\left(\mathbf{W}_{0,c}[1,\mathbf{n},\mathbb{I}_{-1,1}[\mathbf{n}]]\,\omega^*\right),c\right)\right)
\end{aligned}
$$

$$= (\mathsf{tm}\,(\omega^*)\,,\ (\mathsf{bs}\,(\omega^*)\,,c)) = (t,\ (x,c)) = \omega,$$

that was necessary to prove. From (28) it follows that $\mathbb{I}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}})} \in \mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})$. Besides this, in accordance with Remark 6, $\mathbb{I}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}})}[\mathcal{B}] = \mathcal{B}$. Hence, by Property 1(1), we can define the reference frame

$$\mathfrak{l}_0 = \left(\mathbb{I}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}})}, \mathbb{I}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}})}[\mathcal{B}]\right) = \left(\mathbb{I}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}})}, \mathcal{B}\right) \in \mathcal{L}k\,(\mathfrak{KPT}\,(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}))\,.$$

Now, we fix any reference frame $\mathfrak{l} = (U, U[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{KPT}\,(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}))$, where $U = \mathbf{W}_{\lambda,\mathfrak{V}_\mathfrak{f}}\,[1,\mathbf{n},J;\mathbf{a}] \in \mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})$.

According to Properties 1(3, 5), we obtain

$$\mathbb{M}k(\mathfrak{l}) = \mathbb{R}\times\mathfrak{H} = \mathcal{M}\,(\mathfrak{H})\,;$$
$$\langle !\,\mathfrak{l}\leftarrow\mathfrak{l}_0\rangle\,\omega = U\left(\mathbb{I}^{[-1]}_{\mathcal{M}(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}})}\omega\right) = U\omega = \mathbf{W}_{\lambda,\mathfrak{V}_\mathfrak{f}}\,[1,\mathbf{n},J;\mathbf{a}]\,\omega$$
$$(\forall\omega\in\mathbb{B}\mathfrak{s}\,(\mathfrak{l}_0) = \mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right).$$

Using Property 1(3) and equality (9), for an elementary-time state $\omega = (t,(x,c)) \in \mathbb{B}\mathfrak{s}(\mathfrak{l})$ we get

$$\mathbf{Q}^{\langle\mathfrak{l}\rangle}(\omega) = (\mathsf{tm}\,(\omega)\,,\mathbf{q}(\mathsf{bs}\,(\omega))) = (t,\mathbf{q}\,((x,c))) = (t,x) = \omega^*. \tag{29}$$

Hence, using Definition 4 (item 1) and equality (8), we deduce that

$$\begin{aligned}\mathbf{Q}^{\langle\mathfrak{l}\,\leftarrow\,\mathfrak{l}_0\rangle}(\omega) &= \mathbf{Q}^{\langle\mathfrak{l}\rangle}\,(\langle !\,\mathfrak{l}\leftarrow\mathfrak{l}_0\rangle\,\omega) = \left(\mathbf{W}_{\lambda,\mathfrak{V}_\mathfrak{f}}\,[1,\mathbf{n},J;\mathbf{a}]\,\omega\right)^*\\ &= \mathbf{W}_{\lambda,c}\,[1,\mathbf{n},J;\mathbf{a}]\,\omega^* \quad \left(\forall\omega = (t,(x,c))\in\mathbb{B}\mathfrak{s}\,(\mathfrak{l}_0) = \mathbb{B}\mathfrak{s}(\mathcal{B}) \subseteq \mathcal{M}\left(\mathfrak{H}_{\mathfrak{V}_\mathfrak{f}}\right)\right).\end{aligned} \tag{30}$$

2. 2.1. Suppose that there exist elementary states $\widetilde{x}_1 = (x_1,c_1)\,, \widetilde{x}_2 = (x_2,c_2) \in \mathfrak{Bs}(\mathcal{B})$ such that $c_1 \neq c_2$. Since, by formula (1), $\mathfrak{Bs}(\mathcal{B}) = \{\mathsf{bs}\,(\omega)\mid\omega\in\mathbb{B}\mathfrak{s}(\mathcal{B})\}$, there exist elementary-time states of the kind $\omega_1 = (t_1,\widetilde{x}_1) = (t_1,(x_1,c_1)) \in \mathbb{B}\mathfrak{s}(\mathcal{B})$, $\omega_2 = (t_2,\widetilde{x}_2) = (t_2,(x_2,c_2)) \in \mathbb{B}\mathfrak{s}(\mathcal{B})$. According to conditions of Theorem, the set $\widetilde{\mathfrak{V}_\mathfrak{f}}\setminus\{0,\infty\} = \widetilde{\mathfrak{V}_\mathfrak{f}}^{\,\mathsf{fin}+}$ contains at least two elements. So, there exists a positive real number $\widetilde{\lambda}$ such that $\widetilde{\lambda}\in\widetilde{\mathfrak{V}_\mathfrak{f}}$ and $\widetilde{\lambda}\neq\sigma\,(c_1,c_2)$. Now, we consider two cases.

Case 2.1.1: $\omega_1^* \neq \omega_2^*$. Consider any fixed operator $J_1 \in \mathfrak{U}\,(\mathfrak{H}_1)$. By Lemma 1, there exist $\mathbf{n}_1 \in \mathbf{B}_1\,(\mathfrak{H}_1)$ and $\mathbf{a}_1 \in \mathcal{M}\,(\mathfrak{H})$ such that

$$\mathbf{W}_{\widetilde{\lambda},c_1}\,[1,\mathbf{n}_1,J_1;\mathbf{a}_1]\,\omega_1^* = \mathbf{W}_{\widetilde{\lambda},c_2}\,[1,\mathbf{n}_1,J_1;\mathbf{a}_1]\,\omega_2^*. \tag{31}$$

Let us introduce the reference frame

$$\mathfrak{l}_1 := (U_1, U_1[\mathcal{B}]) \in \mathcal{L}k\,(\mathfrak{KPT}\,(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f}))\,,\quad\text{where}$$
$$U_1 := \mathbf{W}_{\widetilde{\lambda};\mathfrak{V}_\mathfrak{f}}\,[1,\mathbf{n}_1,J_1,\mathbf{a}_1] \in \mathfrak{PT}_+(\mathfrak{H};\mathfrak{V}_\mathfrak{f})\,.$$

According to (30) and (31), we get

$$\mathbf{Q}^{\langle\mathfrak{l}_1\,\leftarrow\,\mathfrak{l}_0\rangle}(\omega_1) = \mathbf{W}_{\widetilde{\lambda},c_1}\,[1,\mathbf{n}_1,J_1;\mathbf{a}_1]\,\omega_1^* = \mathbf{W}_{\widetilde{\lambda},c_2}\,[1,\mathbf{n}_1,J_1;\mathbf{a}_1]\,\omega_2^* = \mathbf{Q}^{\langle\mathfrak{l}_1\,\leftarrow\,\mathfrak{l}_0\rangle}(\omega_2)\,.$$

On the other hand, by formula (29), we obtain $\mathbf{Q}^{\langle\mathfrak{l}_0\rangle}(\omega_1) = \omega_1^* \neq \omega_2^* = \mathbf{Q}^{\langle\mathfrak{l}_0\rangle}(\omega_2)$. Thus, for the elementary-time states ω_1,ω_2 we have $\mathbf{Q}^{\langle\mathfrak{l}\,\leftarrow\,\mathfrak{l}_0\rangle}(\omega_1) = \mathbf{Q}^{\langle\mathfrak{l}\,\leftarrow\,\mathfrak{l}_0\rangle}(\omega_2)$, while $\mathbf{Q}^{\langle\mathfrak{l}_0\rangle}(\omega_1) \neq \mathbf{Q}^{\langle\mathfrak{l}_0\rangle}(\omega_2)$. Hence, by Theorem 1, the reference frames $\mathfrak{l}_0$ and $\mathfrak{l}$ do not allow for a universal coordinate transform. Therefore, in accordance with Assertion 1, item 2, the kinematic set $\mathfrak{KPT}\,(\mathfrak{H},\mathcal{B};\mathfrak{V}_\mathfrak{f})$ does not allow for a universal coordinate transform in this case.

Case 2.1.2: $\omega_1^* = \omega_2^*$. Consider any fixed operator $J_2 \in \mathfrak{U}\,(\mathfrak{H}_1)$ and a vector $\mathbf{n}_2 \in \mathbf{B}_1\,(\mathfrak{H}_1)$. According to Corollary 3, there exists $\mathbf{a}_2 \in \mathcal{M}\,(\mathfrak{H})$, such that

$$\mathbf{W}_{\widetilde{\lambda},c_1}\,[1,\mathbf{n}_2,J_2;\mathbf{a}_2]\,\omega_1^* \neq \mathbf{W}_{\widetilde{\lambda},c_2}\,[1,\mathbf{n}_2,J_2;\mathbf{a}_2]\,\omega_2^*. \tag{32}$$

Let us consider the reference frame

$$\mathfrak{l}_2 := (U_2, U_2[\mathcal{B}]) \in \mathcal{L}k\left(\mathfrak{KPT}\left(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_{\mathfrak{f}}\right)\right), \quad \text{where}$$
$$U_2 = \mathbf{W}_{\widetilde{\lambda};\mathfrak{V}_{\mathfrak{f}}}\left[1, \mathbf{n}_2, J_2, \mathbf{a}_2\right] \in \mathfrak{PT}_+\left(\mathfrak{H}; \mathfrak{V}_{\mathfrak{f}}\right).$$

According to (30) and (32), we get

$$\mathbf{Q}^{\langle \mathfrak{l}_2 \leftarrow \mathfrak{l}_0 \rangle}(\omega_1) = \mathbf{W}_{\widetilde{\lambda},c_1}\left[1, \mathbf{n}_2, J_2; \mathbf{a}_2\right]\omega_1^* \neq \mathbf{W}_{\widetilde{\lambda},c_2}\left[1, \mathbf{n}_2, J_2; \mathbf{a}_2\right]\omega_2^* = \mathbf{Q}^{\langle \mathfrak{l}_2 \leftarrow \mathfrak{l}_0 \rangle}(\omega_2).$$

From the other hand, by the formula (29), we obtain $\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega_1) = \omega_1^* = \omega_2^* = \mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega_2)$.

Thus, for the elementary-time states ω_1,ω_2 we have $\mathbf{Q}^{\langle \mathfrak{l}_2 \leftarrow \mathfrak{l}_0 \rangle}(\omega_1) \neq \mathbf{Q}^{\langle \mathfrak{l}_2 \leftarrow \mathfrak{l}_0 \rangle}(\omega_2)$, while $\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega_1) = \mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega_2)$. Hence, by Theorem 1, the reference frames $\mathfrak{l}_0$ and $\mathfrak{l}_2$ do not allow for a universal coordinate transform. Therefore, in accordance with Assertion 1, item 2, the kinematic set $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_{\mathfrak{f}})$ does not allow for a universal coordinate transform.

Thus, if the kinematic set $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_{\mathfrak{f}})$ allows for a universal coordinate transform, then there do not exist elementary states $\widetilde{x}_1 = (x_1, c_1), \widetilde{x}_2 = (x_2, c_2) \in \mathfrak{Bs}(\mathcal{B})$ such that $c_1 \neq c_2$.

2.2. Now we suppose that, in a base changeable set $\mathcal{B}$, there do not exist elementary states $\widetilde{x}_1 = (x_1, c_1), \widetilde{x}_2 = (x_2, c_2) \in \mathfrak{Bs}(\mathcal{B})$ such that $c_1 \neq c_2$. With this assumption, there must be a number $c_0 \in \mathfrak{V}_{\mathfrak{f}}$ such that the arbitrary elementary state $\widetilde{x} \in \mathfrak{Bs}(\mathcal{B})$ can be represented as $\widetilde{x} = (x, c_0)$, where $x \in \mathfrak{H}$. Chose any reference frame

$$\mathfrak{l} := (U, U[\mathcal{B}]) \in \mathcal{L}k\left(\mathfrak{KPT}\left(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_{\mathfrak{f}}\right)\right), \quad \text{where} \quad U = \mathbf{W}_{\lambda;\mathfrak{V}_{\mathfrak{f}}}\left[1, \mathbf{n}, J, \mathbf{a}\right] \in \mathfrak{PT}_+\left(\mathfrak{H}; \mathfrak{V}_{\mathfrak{f}}\right).$$

According to (30), (29), for an arbitrary elementary-time state $\omega = (t, (x, c_0)) \in \mathbb{Bs}(\mathfrak{l}_0) = \mathbb{Bs}(\mathcal{B})$ we obtain

$$\mathbf{Q}^{\langle \mathfrak{l} \leftarrow \mathfrak{l}_0 \rangle}(\omega) = \mathbf{W}_{\lambda,c_0}\left[1, \mathbf{n}, J; \mathbf{a}\right]\omega^* = \mathbf{W}_{\lambda,c_0}\left[1, \mathbf{n}, J; \mathbf{a}\right]\left(\mathbf{Q}^{\langle \mathfrak{l}_0 \rangle}(\omega)\right),$$

where $\mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]$ is a bijection from $\mathcal{M}(\mathfrak{H})$ onto $\mathcal{M}(\mathfrak{H})$ (and, as follows from (29), $\mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]$ is a bijection from $\mathbb{M}k(\mathfrak{l}_0)$ onto $\mathbb{M}k(\mathfrak{l})$). Hence, in accordance with Definition 4, the mapping $\mathbf{W}_{\lambda,c_0}[1, \mathbf{n}, J; \mathbf{a}]$ is a universal coordinate transform from $\mathfrak{l}_0$ to $\mathfrak{l}$. Consequently, the reference frames $\mathfrak{l}_0$ and $\mathfrak{l}$ allow for a universal coordinate transform, i.e., $\mathfrak{l}_0 \rightleftarrows \mathfrak{l}$ (for any reference frame $\mathfrak{l} \in \mathcal{L}k(\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_{\mathfrak{f}}))$). Thus, by Assertion 1, the kinematic set $\mathfrak{KPT}(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_{\mathfrak{f}})$ allows for a universal coordinate transform. □

Similarly to Theorem 5, the following theorem can be proved.

Theorem 6. *Let the set of forbidden velocities* $\mathfrak{V}_{\mathfrak{f}}$ *be such that the corresponding set* $\widetilde{\mathfrak{V}_{\mathfrak{f}}}^{\mathsf{fin}+} = (0, \infty) \setminus \mathfrak{V}_{\mathfrak{f}}$ *contains at least two elements (that is* $\mathbf{card}\left(\widetilde{\mathfrak{V}_{\mathfrak{f}}}^{\mathsf{fin}+}\right) \geq 2$*).*

A kinematic set $\mathfrak{KPT}_0(\mathfrak{H}, \mathcal{B}; \mathfrak{V}_{\mathfrak{f}})$ *allows for a universal coordinate transform if and only if there do not exist elementary states* $\widetilde{x}_1 = (x_1, c_1), \widetilde{x}_2 = (x_2, c_2) \in \mathfrak{Bs}(\mathcal{B})$ *such that* $c_1 \neq c_2$.

Note that results, which is less general then theorems 5 and 6, were announced in the paper [27] (see [27, Theorem 5]) and proved in the preprints [26,29] (see [26, theorems 6 and 7], [29, theorems II.20.1 and II.20.2]).

8. Conclusions

Development of kinematic theories of tachyon movement (which is especially intensified in the recent years) gives rise to a problem of building a new mathematical apparatus that would allow to investigate the evolution of physical systems in the framework of different laws of kinematics. Concerning this problem, in the paper the following results were obtained:

(1) The kinematic sets that represent mathematically strict models of the evolution of physical systems in the framework of kinematics of special relativity theory as well as its tachyon extension based on the generalized Lorentz-Poincare transformations (in the sense of E. Recami and V. Olkhovsky) were constructed.
(2) Also we have constructed kinematic sets that simulate the evolution of physical systems under the condition of hypothesis on existence of particle-dependent velocity of light.
(3) It is proved that the kinematic sets of the first type allow for a universal coordinate transform, whereas the kinematic sets of the second type do not allow for a universal universal coordinate transform in non-trivial cases.

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