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MYROSLAV LVOVYCH GORBACHUK (TO HIS 80TH BIRTHDAY)

Myroslav Lvovych Gorbachuk is our history and, at the same time, our contemporaneity. It seems quite recently his students, post-graduates, colleagues attended his sapid lectures, were onlookers of his inexhaustible energy, tireless creative search, variety of interests, active participation in public life. Even in his last years, being seriously ill, he continued to work in mathematics, to give reports at seminars, conferences, lectures, where he shared with listeners not only his own ideas and results, but the cognizance about famous scientists such as, for instance, N. Abel, L. Euler, D. Hilbert, K. Gauss, J. Lagrange, B. Riemann, R. Weierstrass and others, but the exceptional attention was given to the prominent Ukrainian mathematicians S. Banach, V. Bogolyubov, M. Kravchuk, M. Krein, Ya. Lopatinsky, M. Ostrogradsky, Yu. Sokolov, G. Voroniy etc. In every his lecture or report there was something interesting about mathematics an mathematicians. He even had luck to find and describe some moments of intercourse between M. Ostrogradsky and T. Shevchenko. Myroslav Lvovych sincerely loved his fatherland, its beautiful people, culture, language. He often repeated the sacred for him the academician Kravchuk's words: "My love is mathematics and Ukraine". By his works, he aspired to attract as much as possible attention to the Ukrainian mathematics. He always taught his students in Ukrainian even in the soviet times, when it was not "in fashion" to speak it and the then regime made everything for Ukrainians to forget their native language as fast as possible.

Mathematics was a tsarina of sciences for Myroslav Lvovych. One of his works, devoted to G. Voroniy's creation and published in the book "Axioms for descendants", just so is called: "On the top of a tsarina of sciences".

M.L. Gorbachuk was born on March 8, 1938, in Rybotychi, Przemyśl region (now a part of Poland), in a peasant family with seven children. To keep the family, their father Lev Dmytrovych was also engaged in shoemaking. The family was religious, they confessed the Greek-Catholicism. After the World War II, in connection with mass forced evictions of Ukrainians out from Lemkovshchyna and Kholmshchyna, the family left its fatherland for Western Ukraine. As a result they found themselves in the village P'yanovychi, Lviv region. The family had nowhere to live. Watching this, the local priest took pity on them and sheltered in his home. In spite of all difficulties of the postwar life, the parents devoted much of attention to upbringing of the children. The Bible and music were profoundly respected in this house. Lev Dmytrovych was a self-taught violinist. Together with other musicians he played in rural evening parties, weddings, holy days. This helped the family "to make both ends meet". Myroslav's mother Mykhailyna Ivanivna had a wonderful voice. The boy sang very nice, too. The parents attracted their children to musical art from the early years. Together with them, under accompaniment of violin, the children were singing popular folk songs, Christmas carols, the Sitch riflemen songs.

Father dreamt for his children to become musicians. After a seven year school, under father's influence, Myroslav tried to enter the Drohobych musical teaching institution. However it was not his fortune to be a musician (he did not cope with a Russian dictation). But as the saying is, " there is some good even in misfortune". So, he continued education at the secondary school located in the neighboring village Biskovychi. Every day, to get to school and come back home the boy overcame on foot the distance about fifteen kilometers. Studying at this school, he understood that mathematics was his favourite subject and he would never abandon it. Having graduated from the school, he entered in 1956 the I. Franko University of Lviv to study this subject at the Mechanics and Mathematics Faculty.

As is well-known, this university played an important role in development of mathematics in the 20th century. The level of teaching it there was high enough. In 1920-1930, it was formed a group of talanted mathematicians such as, for instance, S. Banach, G. Steinhaus, Yu. Shauder, S. Mathur and others which elaborated the fundamental principles of functional analysis, and this affected essentially on the level of teaching this branch of mathematics at the University. The youth was deeply impressed by lectures of I.G. Sokolov and Ya.B. Lopatynsky which left an indelible mark onto all his further scientific life. A lot of students of the Lviv mathematical school became well-known scientists, heads of mathematical collectives, founders of new directions in mathematics. The academicians of NAS in mathematics O. Parasyuk, I. Skrypnik, associate members I. Danilyuk, B. Pshenychny. Yu. Trohimchuk, V. Lyantse, A. Holdberg, V. Skorobohatko, B. Ptashnyk and, of course, M. Gorbachuk are among them.

The graduation work of M.L. Gorbachuk was guided by V.E. Lyantse, an excellent mathematician and highly intelligent person. It was V.E. Lyantse who recommended him for the post-graduate education at the Institute of Mathematics of Academy of Sciences of USSR in Kyiv, and on his good advise, the youth chose the Mathematical Analysis Department of this Institute and in 1961, became its graduate student. The education was guided by the Department Head Yu.M. Berezansky, highly skilled and honest man. Under his influence and support, the initial and subsequent M.L. Gorbachuk's scientific interests were formed. An important role was played also by M.G. Krein's works which Myroslav studied with pleasure being still a student. The first personal meeting with him took place at Krein's seminar in Odessa which was one of the main centers of mathematical life not only in Ukraine but in other countries. The second unforgotten one evetuated not far from T.G. Shevchenko memorial, at the First (in the USSR) Kaniv Summer Mathematical School (1963), where he gave some lectures on the theory of perturbations of self-adjoint operators with continuous spectrum. Here Mark Grigorovych blessed the marriage of Myroslav Gorbachuk and Valentyna Plushchova (since Gorbachuk), the Yu. Berezansky's research student, too.

When studying in post-graduate course, Myroslav attended frequently the meetings of Ukrainian intelligentsia, among which such its representatives as I. Drach, V. Symonenko, Ye. Sverstuyk and others were present. He didn't forget music either and with pleasure attended Kyiv opera-house and Philarmonic Society.

After research studentship, the whole scientific activity of M.L. Gorbachuk was going within the Institute's of Mathematics walls. Here he obtained both the Candidate/PhD (1965) and the Doctor of Science (1973) degrees in Mathematics for the theses "Positive definite operator-valued functions" and "Some problems of the spectral theory of differential operators in spaces of vector-valued functions", respectively (the scientific adviser was Yu.M. Berezansky), here he held the positions of a junior and a senior scientific researchers at the Mathematical Analysis Department, a chief of the Partial Differential Equations Laboratory in this Department, in 1986-2016 he headed the Department of the same name and since 2016 he was a chief researcher at the Non-Linear Differential Equations Department. In 2000, he was elected an associate member of the National Academy of Sciences of Ukraine.

Simultaneously M. Gorbachuk taught at the T. Shevchenko University and the National Technic University "KPI" of Kyiv. His rich in content and understandable lectures always attracted attention of talanted students. Under his guidance a number of them defended their dissertations. There are 7 Doctors of Sciences and 29 Candidates working in Ukraine and abroad. Here they are: A. Kochubei (1987), V. Mikhailets (1989), Do Kong Khan (1991), L. Vainerman (1991), O. Reznikov (1994), V. Gorodetsky (1995), S. Kuzhel (2002) (Doctors of Sciences) and L. Vainerman (1974), V. Mikhailets (1975), A. Kochubei (1977), V. Kutovyi (1979), M. Rybak (1980), O. Kashpirovsky (1981), N. Laptyeva (1983), V. Levchuk (1984), L. Fedorova (1984), V. Gorodetsky (1985), A. Knyazyuk (1985), P. Dudnikov (1986), I. Izvekov (1986), B. Knyukh (1986), L. Oliynik (1986), M. Pivtorak (1986), I. Fishman (1987), I. Fedak (1988), Z. Ismailov (1989), S. Kuzhel (1990), O. Shklyar (1990), Yu. Mytnik (1992), M. Bondarenko (1994), M. Markin (1994), O. Martynenko (1994), O. Butyrin (1995), Ya. Grushka (2000), Yu. Linchuk (2007), S. Torba (2008) (Candidates of Sciences).

The mathematical works of M.L. Gorbachuk include 3 monographs and about 200 papers which are of high level and quality, and broad in topics. They concern various problems of functional analysis, operator theory, theories of generalized functions, abstract differential equations, approximation of functions, mathematical physics, history of mathematics. Some of these works opened certain directions in these branches. His achievements are recognized by leading specialists, and they became an initial moment for obtaining new deep results by his followers. Some of his works were included into the series "New methods in the theory of generalized functions and their applications to mathematical physics" honoured with the State Prize of Ukraine (1998). He was also the first who obtained the Krein Prize founded in 2008 by National Academy of Sciences for the works relating to the spectral theory of operators and its applications. The attainment in the theory of smooth and generalized vectors of a closed operator in a Banach space and their applications were awarded with Krylov Prize (1994).

The mathematical creation of M.L. Gorbachuk distinguishes itself by the tendency of penetrating into the historical aspects of development of every subject and searching for something common in, at first sight, too different problems, and finding a general approach that gives a possibility not only to look from a single point of view at these problems but to extend the sphere of problems where this approach might be applied, and even sometimes to perfect the well-known results.

The main M.L. Gorbachuks's scientific interests are concentrated around the following directions: spectral theory of operators, solvability problems for differential equations in a Banach space in various classes of vector-valued functions, initial and boundary value problems for operator differential equations, theory of non-selfadjoint operators, generalized functions and their applications, approximation theory, theory of semigroups of linear operators, harmonic analysis, problems of hydrodynamics. His first works were devoted to investigation of positive definite kernels whose values are operators in a Hilbert space. The representations in terms of eigenfunctions of an ordinary differential equation were described for them. In the case of a positive definite function given on a finite interval, he developed the theory of its extensions to the whole axis analogous to that of M. Krein's in the scalar one. Moreover, the most extreme, so called totally indefinite case of nonuniqueness of an extension (analog of a totally undetermined classical moment problem) was considered. This made it possible to approach from a new point of view to description of all positive definite extensions for a positive definite function of two variables from a finite restungle or strip to the whole plane. These results are closely connected with theories of entire operators (M.G. Krein) and eigenfunction expansions for self-adjoint operators (Yu.M. Berezansky).

In the spectral theory of operators M.L. Gorbachuk investigated the domains of minimal and maximal operators generated by a fist- and a second-order differential equations on a finite interval, whose coefficients are unbounded operators in a Hilbert space, described in terms of boundary conditions various classes of extensions (maximal dissipative, accretive and others) of the minimal operator and studied their spectral properties. In the case of bounded operator coefficients, such questions were considered by F. Rofe-Beketov. Under this condition, the situation is analogue to that described by M. Krein for ordirary differential expressions. As for partial differential ones, it is more complicated because the functions from the maximal operator domain are smooth enough inside their domains of definition and their boundary values may not exist in usual sense, they exist in certain classes of generalized functions. Thus, in the presence of an unbounded coefficient, it should not be expected full similarity with the case of ordinary differential equations. So, the problem arised of finding the boundary conditions corresponding to self-adjoint extensions of the minimal operator and explaining whether all self-adjoint extensions are determined by these bondary conditions which is of interest in the theory of boundary value problems. First this problem was considered by M.G. Krein for an ordinary formally self-adjoint expression on the base of general theory of extensions for symmetric operators. Its application to partial differential equations is complicated in connection with infiniteness of the deficiency index of the minimal operator. An essential step to overcome these difficulties was made in the works of F.S. Rofe-Beketov, where in a compact binary (linear) relation form, all the self-adjoint extensions of minimal operator with infinite defect numbers, generated by an ordinary differential expression on a finite interval were described.

Using the binary relation technology, M.L. Gorbachuk solved the above problems for the Sturm-Liouville equation with unbounded operator potential. The perfect description of all maximal dissipative (in partiqular, self-adjoint) and some other boundary value problems was obtained and their spectral properties were studied. Soon the results were extended by him and his disciples to high-order differential operators and later, on an arbitrary Hermitian operator. The main aim was to develope the theory of extensions in terms of abstract boundary conditions. Imposing various restrictions on the operators in a binary relation, corresponding to an extension (in applications, this is equivalent to choice of a specific class of boundary value problems), the spectral properties of the extensions were investigated, too. Here is M. Krein's opinion on the M.L. Gorbachuk's results in this direction: "Myroslav Lvovych obtained at first the description and gave a deep analysis of boundary conditions for differential operations with unbounded operator coefficient in a space of vector-valued functions, which cover important classes of partial differential operations. He succeeded in realizing this analysis due to his high level mathematical culture and ingenuity, by involving a big arsenal of means from the abstract operator theory (rigged Hilbert spaces, generalized resolvents, entire Hermitian operators with infinite defect numbers. a scaled space of generalized elements, the directing functional metod). He investigated a number of questions which in the boundary value problems were avoided at all or marked as very difficult and unsolvable".

One of the most important achievements of M.L. Gorbachuk is the theory of generalized functions constructed by using an arbitrary closed linear operator in a Banach space instead of the differentiation one as it was usually done (for instance, in the space of square-integrable functions). On the base of it, the abstract variant of Paley-Wiener theorem was obtained. This theory made it possible to study the structure of solutions to various types and orders differential equations on $(0, \infty)$ in a Banach space with unbounded operator coefficients, describe them inside this interval and investigate their boundary values at 0. Moreover, the conditions were found under which a solution can be extended to an entire vector-valued function of a certain finite order and a finite type, and it was shown that an analog of Phragmen-Lindelöf principle was realized for such solutions. The above theory contains, in particular, a considerable part of results on boundary values of analytic, harmonic, polyharmonic and other functions (for example, Fatou, Komatsu, Riesz theorems). The Fourier series expansions in generalized eigenvectors of the basic operator and the localization principle proved by him allowed to justify some statements of mathematical physics.

The approach developed by M. Gorbachuk admitted also to consider from operator point of view the problem of correct solvability in certain classes of vector-valued functions of some initial and boundary value problems (Dirichlet, Cauchy, Neumann) for operator differential equations with unbounded operator coefficients in a Banach space. In such a way, the problem of locally analytic solvability of the Cauchy problen for a system of partial differential equations, whose coefficients and initial data are locally analytic functions, was considered. Earlier S. Kovalevskaya showed that, as distinguished of the case of ordinary differential equations, the locally solvability of this system held not always. Using the operator approach Myroslav Lvovych explained in detail the reason of this divergence: the point is that not each analytic in a neighborhood of 0 function is an analytic vector of the operator associated with the problem under consideration. The necessary and sufficient conditions on initial data for such a problem to be solvable were found by him.

Myroslav Lvovych investigated also the Cauchy problem for differential equations in a Banach space over the non-Archimedean field of p-adic numbers. He found a criterion for its well-posedness in the class of locally analytic vector-valued functions and showed how the Cauchy-Kowalewskaya theorem for p-adic partial differential equations can be obtained as a special case of this criterion.

A part of M.L. Gorbachuk's results is devoted to studying behavior of solutions of differential equations on $(0, \infty)$ in a Banach space when approaching to infinity and their stability which is important for applications to problems of hydrodynamics. The conditions, necessary and sufficient for such an equation to be uniformly, uniformly exponentially or uniformly but not uniformly exponentially stable were presented. These results generalize the corresponding assertions of Datko, M. Krein and Pazy. Moreover, the relation between the decrease degree of a stable solution and the properties of its initial data was established.

M.L. Gorbachuk has also found a universal (operator)approach to problems of approximation theory which embraces a lot of well-known and new concrete problems of approximation of functions. This approach enabled not only to obtain from the unique point of view a number of classical direct and inverse theorems of this theory, but to widen their class. The one-to-one correspondence between the smoothness degree of a vector in a Banach space with respect to the given closed operator and the degree of convergence to 0 of its best approximation by vectors of exponential type of this operator as well as between smoothness degree of a weak solution of a first order differential equation in a Hilbert space and the degree of convergence to 0 of its best approximation by entire solutions of exponential type was established. The exact a priori estimates of approximation error for a solution of an operator equation using the variational methods (Riesz, of least squares etc) were obtained, too. It should be noted that in applications, the direct and inverse theorems are used, as a rule, for equations in a Banach space, however their proof is considerably simpler in a Hilbert one. Myroslav Lvovych showed how these theorems could be reformulated in the case of a Banach space rigged by Hilbert spaces with positive and negative norms.

It is well-known that the basic mathematical instrument when studying abstract differential equations is the theory of semigroups. Giving his opinion on its role in mathematics, E. Hille wrote: "I greet a semigroup wherever I meet it, but we come across it everywhere". One of the most important problems in mathematical analysis lies in constructing the exponential function of a closed operator A in a Banach space $\mathfrak{B}$. The theory of semigroups solves this problem in the case where A is the generator of a strongly continuous semigroup which is equivalent to well-posedness of the Cauchy problem for equation $y'(t) = Ay(t)$, and if the operator A is bounded, its solutions are described by the exponential function of A which can be presented as a power series or an exponential limit from A. But if the operator A is unbounded, then the question arises just what is required to understand under e^{tA} in the case of an arbitrary group or semigroup. In thirtieth of last century A.M. Kolmogorov set up the problem of existence of a maximal dense in $\mathfrak{B}$ subspace $\mathfrak{B}_1$ on whose elements x, $e^{tA}x$ can be represented in the form of a power series. Similarly, in 1946 E. Hille raised the question of finding an analogous subspace $\mathfrak{B}_2$ on which $e^{tA}x$ is the exponential limit of A. The Kolmogorov's problem was solved by I.M. Gelfand (1939) for a bounded group. M.L. Gorbachuk succeeded in solving both Kolmogorov's and Hille's problems for any C_0-group and even for an analytic semigroup. Morever, it was shown by him that the subspaces $\mathfrak{B}_1$ and $\mathfrak{B}_2$ were nothing else but the space of entire vectors of the generator A. The result concerning Hille's problem was generalized to an arbitrary closed operator A. Thus, Myroslav Lvovych proposed the way of restoration of a C_0-semigroup directly by its generator, not some functions (sometimes complicated enough) from it, which embarrassed the renewal process.

To find the conditions on initial data, necessary and sufficient for the Cauchy problem for a general operator differential equation to be solved in certain classes of vector-valued functions (locally analytic, entire, entire of finite order and finite type and others) M.L. Gorbachuk developed the theory of restrictions and extensions of analytic semigroups which was taken as a basis for study of boundary value problems for abstract differential equations.

Except of direct scientific and pedagogical activity, Myroslav Lvovych participated actively in the scientific public work as a President of the Kyiv (1993-2006) and then Ukrainian (2005-2012) Mathematical Societies. He was a member of Editorial Boards of the "Ukrainian Mathematical Journal", and the journals "Methods of Functional Analysis and Topology" and "In the World of Mathematics", one of the heads of the well-known Kyiv Seminar on Functional Analysis.

M.L. Gorbachuk was not only a brilliant mathematician, who has solved a lot of difficult problems that initiated some important directions and looked for a deep connection between them. He was a kind, responsive person, a careful son for his parents, decent family man, a genuine patriot of Ukraine. He never left his fatherland although he had all the possibilities to move to the USA (his mother was a native of it), but he didn't do this, and he asked also his talented students not to leave their country in search of a better life. Myroslav Lvovych firmly believed that even the darkest night was changed into a day-break, and the worthy of world respect Ukraine would be built up. And he was doing his utmost for this as its scientist and its citizen. Myroslav lvovych died on January 8, 2017. Eternal memory to him.

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