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A CLASS OF REPRESENTATIONS OF C^*-ALGEBRA GENERATED BY q_{ij}-COMMUTING ISOMETRIES

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Abstract. For a C^*-algebra generated by a finite family of isometries s_j, $j = 1, \dots, d$, satisfying the q_{ij}-commutation relations

$$s_j^* s_j = I, \quad s_j^* s_k = q_{ij} s_k s_j^*, \qquad q_{ij} = \bar{q}_{ji}, |q_{ij}| < 1, \ 1 \le i \ne j \le d,$$

we construct an infinite family of unitarily non-equivalent irreducible representations. These representations are deformations of a corresponding class of representations of the Cuntz algebra $\mathcal{O}_d$.

Для C^*-алгебри, породженої скінченною сім'єю ізометрій s_j, $j = 1, \dots, d$, що задовольняє q_{ij}-комутаційним співвідношенням

$$s_j^* s_j = I, \quad s_j^* s_k = q_{ij} s_k s_j^*, \qquad q_{ij} = \bar{q}_{ji}, |q_{ij}| < 1, \ 1 \le i \ne j \le d,$$

ми будуємо нескінченну сім'ю унітарно нееквівалентних незвідних представлень. Ці представлення є деформаціями відповідного класу представлень алгебри Кунца $\mathcal{O}_d$.

1. Introduction

Algebras with Wick ordering [1, 5, 4] and their representations have been studied by various authors. A Wick algebra $W_d(T)$ is an associative unital $*$-algebra over $\mathbb{C}$ generated by a finite number of elements a_j, a_j^*, $j = 1. \dots, d$, for which the following relations hold:

$$a_j^* a_k = \delta_{jk} I + \sum_{l,m=1}^{d} T_{jk}^{lm} a_m a_l^*, \quad \bar{T}_{jk}^{lm} = T_{kj}^{ml}, \qquad j, k, l, m = 1, \dots, d.$$

In particular, if all $T_{jk}^{lm} = 0$, then the relations in $W_d(T)$ generate the Cuntz-Toeplitz C^*-algebra $\mathcal{O}_d^0$,

$$a_j^* a_k = \delta_{jk} I, \qquad j, k = 1, \dots, d.$$

It was cojectured (and proved in some cases) in [4] that for sufficiently small coefficients T_{jk}^{lm}, $j, k, l, m = 1, \dots, d$, for $W_d(T)$ there exists a universal enveloping C^*-algebra which is isomorphic to $\mathcal{O}_d^0$. In [3], it has been shown that the C^*-algebra generated by a pair of q-commuting isometries,

$$s_1^* s_1 = s_2^* s_2 = I, \qquad s_1^* s_2 = q s_2 s_1^*,$$

is isomorphic to $\mathcal{O}_2^0$ for all $|q| < 1$; however, in the case where $d > 2$, such a result is still a conjecture.

In this paper, we study representations of a Wick ordered C^*-algebra W generated by elements s_j, $j = 1, \dots, d$, satisfying the relations

$$s_i^* s_i = I, \quad s_i^* s_j = q_{ij} s_j s_i^*, \quad |q_{ij}| < 1, \quad q_{ij} = \bar{q}_{ji}, \qquad 1 \le i \ne j \le d. \tag{1.1}$$

In a general situation of Wick algebras, the most known is the Fock representation [2, 7] and coherent [6] representations. We construct a new family of irreducible non-Fock representations which are deformations of a certain family of irreducible representations of the Cuntz algebra $\mathcal{O}_d$.

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2. Preliminaries

2.1. **Some notations.** We start with some notations. Let $\alpha = (\alpha_1, \dots, \alpha_m) \in \{1, \dots, d\}^m$ be a finite multiindex of length m, $|\alpha| = m$, let $\Lambda_m = \{1, \dots, d\}^m$ be the set of all finite multiindices of length m, $\Lambda_0 = \emptyset$, and let $\Lambda^0 = \cup_{m=0}^{\infty} \Lambda_m$ be the set of all finite multiindices of arbitrary length. Also, we will use the set $\Lambda = \{1, \dots, d\}^\infty$ of all infinite multiindices. For each finite multiindex $\alpha = (\alpha_1, \dots, \alpha_m) \in \Lambda^0$ we use notation $s_\alpha = s_{\alpha_1} \dots s_{\alpha_m}$. For a finite multiindex we use standard mappings:

$$\Lambda_m \ni \alpha = (\alpha_1, \dots, \alpha_m) \mapsto \sigma(\alpha) = (\alpha_2, \dots, \alpha_m) \in \Lambda_{m-1},$$
$$\Lambda_m \ni \alpha = (\alpha_1, \dots, \alpha_m) \mapsto \sigma_j(\alpha) = (j, \alpha_2, \dots, \alpha_m) \in \Lambda_{m+1}, \quad j = 1, \dots, d.$$

If α does not contain j, then (1.1) implies

$$s_j^* s_\alpha = q(j, \alpha) s_\alpha s_j^*, \quad q(j, \alpha) = q_{j\alpha_1} \dots q_{j\alpha_m}.$$

If α contains j, then α can be represented as $\alpha = (\alpha' j \alpha'')$, where α' does not contain j, and we have

$$s_j^* s_\alpha = q(j, \alpha') s_{\alpha'} s_{\alpha''} = q(j, \alpha) s_{\alpha \setminus j}$$

(here and below, we denote by $\alpha \setminus j = (\alpha' \alpha'')$ the multiindex obtained from α by removing the first occurrence of j, and set $q(j, \alpha) = q(j, \alpha')$ for convenience).

Similarly, for any finite multiindices $\alpha = (\alpha_1, \dots, \alpha_m) \in \{1, \dots, d\}^m$, $\beta = (\beta_1, \dots, \beta_n) \in \{1, \dots, d\}^n$, $n, m \ge 1$, one can define $\alpha \setminus \beta$ inductively as follows:

$$\alpha \setminus \beta = \begin{cases} (\alpha \setminus \beta_1) \setminus \sigma(\beta), & \beta_1 \subset \alpha, \\ \alpha \setminus \sigma(\beta), & \text{otherwise.} \end{cases}$$

Setting $s_\emptyset = I$, we obtain

$$s_\alpha^* s_\beta = q(\alpha, \beta) s_{\beta \setminus \alpha} s_{\alpha \setminus \beta}^*,$$

where $q(\alpha, \beta)$ is calculated in an obvious way inductively.

If β is a permutation of α, then $\alpha \setminus \beta = \beta \setminus \alpha = \emptyset$, and we have $s_\alpha^* s_\beta = q(\alpha, \beta) I$. Also, in this case, for any multiindex δ we have

$$s_{(\alpha\delta)}^* s_{(\beta\delta)} = q(\alpha, \beta) I = q((\alpha\delta), (\beta\delta)) I.$$

For infinite multiindices, we define

$$q(\alpha, \beta) = \lim_{m \to \infty} q((\alpha_1, \dots, \alpha_m), (\beta_1, \dots, \beta_m)). \tag{2.2}$$

The limit exists, since $|q| < 1$ and is nonzero only if the sequence becomes stationary, i.e., if there exists m such that $\sigma^m(\alpha) = \sigma^m(\beta)$, and $(\beta_1, \dots, \beta_m)$ is a permutation of $(\alpha_1, \dots, \alpha_m)$.

2.2. **Fock representation.** Fock representation π_F is a $*$-representation of W which is determined by the condition that there exists a (unique up to a constant) unit vector Ω for which $\pi_F(s_j^*)\Omega = 0$, $1 \le j \le d$. The representation space $\mathcal{F}$ (the Fock space) is spanned by the vectors $e_\alpha = \pi_F(s_\alpha)\Omega$, $\alpha \in \Lambda^0$, and equipped with the Fock scalar product

$$(e_\alpha, e_\beta)_F = (\pi_F(s_\beta^* s_a \alpha)\Omega, \Omega)_F = \begin{cases} q(\beta, \alpha), & |\alpha| = |\beta|, \ \beta \text{ is a permutation of } \alpha, \\ 0, & \text{otherwise.} \end{cases}$$

This scalar product is known to be positive [1]. In particular, the finite-dimensional subspaces $\mathcal{F}_n$ spanned by e_α, $\alpha \in \Lambda_n$, $n \ge 0$, are orthogonal to each other, therefore,

$$\mathcal{F} = \bigoplus_{n=0}^{\infty} \mathcal{F}_n.$$

Operators of the Fock representation are defined as follows

$$\pi_F(s_j)e_\alpha = e_{\sigma_j(\alpha)},$$
$$\pi_F(s_j^*)e_\alpha = q(j,\alpha)\pi_F(s_{\alpha\setminus j})\pi_F(s_{j\setminus\alpha}^*)\Omega = \begin{cases} q(j,\alpha)e_{\alpha\setminus j}, & \alpha \text{ contains } j,\\ 0 & \text{otherwise.}\end{cases}$$

3. A construction of non-Fock representations

We start with introducing an appropriate Hilbert space. Consider an (uncountable) set of vectors e_γ, $\gamma\in\Lambda$. For these vectors, define

$$(e_\beta, e_\gamma) = q(\gamma,\beta), \tag{3.3}$$

where $q(\beta,\gamma)$ was defined above in (2.2). In particular, for any $\beta\in\Lambda$ we have $(e_\beta,e_\beta)=1$.

We say that infinite multiindices $\alpha,\beta\in\Lambda$ are equivalent, denoted by $\beta\sim\alpha$, if they "have the same tails up to a shift", i.e., there exist numbers m,n, such that $\sigma^m(\beta)=\sigma^n(\gamma)$. Fix an infinite multiindex α and consider a countable family of vectors $(e_\beta \mid \beta\sim\alpha)$. Define $\tilde H_\alpha$ as a linear span of this family.

Proposition 3.1. *Form* (3.3) *is well-defined and positive on* $\tilde H_\alpha$.

Proof. Fix a sequence $\Lambda\ni\alpha=(\alpha_1,\alpha_2,\dots)$, and define operators $J_k\colon \mathcal F_k\to\mathcal F_{k+1}$, $k=0,1,\dots$, as follows:

$$\mathcal F_k \ni e_{(\gamma_1,\dots,\gamma_k)} \mapsto J_k e_{(\gamma_1,\dots,\gamma_k)} = e_{(\gamma_1,\dots,\gamma_k,\alpha_{k+1})}\in\mathcal F_{k+1},\quad k=0,1,\dots,$$

and extend this action to the whole $\mathcal F_k$ by linearity. These operators are well-defined since $(e_\gamma)_{\gamma\in\Lambda_k}$ form a linear basis in $\mathcal F_k$.

Lemma 3.2. *The operator* J_k*,* $k\ge 0$*, is an isometric embedding of* $\mathcal F_k$ *into* $\mathcal F_{k+1}$*.*

Proof. Take $\beta,\gamma\in\Lambda_k$, then

$$\begin{aligned}(J_k e_\beta, J_k e_\gamma)_{\mathcal F_{k+1}} &= (e_{(\beta_1,\dots,\beta_k,\alpha_{k+1})}, e_{(\gamma_1,\dots,\gamma_k,\alpha_{k+1})})\\ &= (\pi_F(s_\gamma^* s_\beta)e_{\alpha_{k+1}}, e_{\alpha_{k+1}}) = q(\gamma,\beta)(\pi_F(s_{\beta\setminus\gamma}s_{\gamma\setminus\beta}^*)e_{\alpha_{k+1}}, e_{\alpha_{k+1}})\\ &= q(\gamma,\beta)(\pi_F(s_{\gamma\setminus\beta})^*e_{\alpha_{k+1}}, \pi_F(s_{\beta\setminus\gamma})^*e_{\alpha_{k+1}}) = (e_\beta,e_\gamma)_{\mathcal F_k}.\end{aligned}$$

Indeed, if γ is a permutation of β, then $\beta\setminus\gamma=\gamma\setminus\beta=\emptyset$, and since $s_\emptyset=I$,

$$\begin{aligned}q(\gamma,\beta)(\pi_F(s_{\gamma\setminus\beta})^*e_{\alpha_{k+1}}, \pi_F(s_{\beta\setminus\gamma})^*e_{\alpha_{k+1}}) &= q(\gamma,\beta)(e_{\alpha_{k+1}},e_{\alpha_{k+1}})\\ &= q(\gamma,\beta) = (e_\beta,e_\gamma)_{\mathcal F_k}.\end{aligned}$$

If γ is not a permutation of β, then $\beta\setminus\gamma$ and $\gamma\setminus\beta$ are non-empty and disjoint. Since $\pi_F(s_j)^*e_{\alpha_{k+1}}\ne 0$ only if $j=\alpha_{k+1}$, this implies that

$$q(\gamma,\beta)(\pi_F(s_{\gamma\setminus\beta})^*e_{\alpha_{k+1}}, \pi_F(s_{\beta\setminus\gamma})^*e_{\alpha_{k+1}}) = 0 = (e_\beta,e_\gamma)_{\mathcal F_k}. \qquad \square$$

Consider an inductive limit $\tilde H_\alpha^0 = \varinjlim_{J_k}\mathcal F_k$. This space can be naturally identified with a span of the vectors e_β, $\beta\in\Lambda$, over all β for which there exists m such that $\beta_k=\alpha_k$ for $k>m$. Lemma 3.2 and the positivity of the Fock scalar product yields that

$$(e_\beta,e_\gamma) = (e_{(\beta_1,\dots,\beta_k)}, e_{(\gamma_1,\dots,\gamma_k)}),\quad \text{if } \beta_j=\gamma_j=\alpha_j \text{ for all } j>k,$$

is a well-defined positive form on $\tilde H_\alpha^0$, and that

$$\mathcal F_k\ni e_{(\gamma_1,\dots,\gamma_k)}\mapsto e_{(\gamma_1,\dots,\gamma_k,\alpha_{k+1},\alpha_{k+2},\dots)}\in\tilde H_\alpha^0$$

is an isometric embedding.

Obviously, $\tilde H_\alpha^0\subset\tilde H_\alpha$, but in general (unless α is equivalent to a stationary sequence) it is a proper subset. Let $\beta\sim\alpha$. The same way, consider space $\tilde H_\beta^0$ with the corresponding scalar product. If there exists m such that $\beta_k=\alpha_k$ for all $k>m$ (or equivalently,

$\sigma^m(\alpha) = \sigma^m(\beta)$), then $\tilde{H}^0_\alpha = \tilde{H}^0_\beta$. Otherwise α and β differ in an infinite set of indices (so that $q(\alpha, \beta) = 0$) and we set $\tilde{H}^0_\alpha$ and $\tilde{H}^0_\beta$ to be orthogonal. Similarly, for any $\beta \sim \gamma \sim \alpha$ we define

$$(e_\beta, e_\gamma) = \begin{cases} (e_{(\beta_1,\dots,\beta_m)}, e_{(\gamma_1,\dots,\gamma_m)})_{\mathcal{F}_m}, & \text{there exists } m, \text{ for which } \sigma^m(\beta) = \sigma^m(\gamma), \\ 0, & \text{otherwise.} \end{cases}$$

The arguments above show that this form is well-defined and positive on the whole $\tilde{H}_\alpha$. One can easily see that the latter expression is equal to $q(\gamma, \beta)$. □

Define H_α as a completion of $\tilde{H}_\alpha$ with respect to the introduced above scalar product.

Theorem 3.3. *1. Operators in H_α,*

$$\pi_\alpha(s_j)e_\beta = e_{\sigma_j(\beta)}, \quad \pi_\alpha(s_j^*)e_\beta = \begin{cases} 0, & \beta \text{ does not contain } j, \\ q(j, \beta)e_{\beta\setminus j}, & \text{otherwise,} \end{cases}$$

form well-defined $*$-representation of the C^*-algebra W.

2. This representation is irreducible.

3. Representations corresponding to multiindices α, α' are unitary equivalent iff the corresponding Hilbert spaces coincide, i.e., $\alpha \sim \alpha'$.

4. The representation π_α is not unitary equivalent to the Fock representation.

Proof. 1. We need to verify that $\pi_\alpha(s_j^*) = \pi_\alpha(s_j)^*$, $j = 1, \dots, d$, and that relations (1.1) hold. Obviously, it is sufficient to verify this on the vectors e_γ, $\gamma \sim \alpha$.

Conditions $\pi_\alpha(s_j^*)e_\beta = \pi_\alpha(s_j)^* e_\beta$, $j = 1, \dots, d$, hold due to the way the scalar product is constructed. Indeed,

$$(\pi_\alpha(s_j)^* e_\beta, e_\gamma) = (e_\beta, \pi_\alpha(s_j)e_\gamma) = (e_\beta, e_{\sigma_j(\gamma)}),$$

$$(\pi_\alpha(s_j^*)e_\beta, e_\gamma) = \begin{cases} 0, & \beta \text{ does not contain } j, \\ q(j, \beta)(e_{\beta\setminus j}, e_\gamma), & \text{otherwise.} \end{cases}$$

If β does not contain j, then the both expressions are zero. Assume β contains j. According to the definition of the scalar product, $(e_{\beta\setminus j}, e_\gamma) \neq 0$ only if there exists m_1, for which $\sigma^{m_1}(\beta \setminus j) = \sigma^{m_1}(\gamma)$. Then there exists m_2, for which $\sigma^{m_2}(\beta) = \sigma^{m_2}(\sigma_j(\gamma))$. Take $m = \max(m_1, m_2)$, this will in particular ensure that $(\beta_1, \dots, \beta_{m+1})$ contains j. Then, since the Fock representation is a $*$-representation,

$$\begin{aligned}(e_\beta, e_{\sigma_j(\gamma)}) &= (e_{(\beta_1,\dots,\beta_{m+1})}, e_{(j,\gamma_1,\dots,\gamma_m)})_{\mathcal{F}_{m+1}} \\ &= (e_{(\beta_1,\dots,\beta_{m+1})}, \pi_F(s_j)e_{(\gamma_1,\dots,\gamma_m)})_{\mathcal{F}_{m+1}} \\ &= (\pi_F(s_j^*)e_{(\beta_1,\dots,\beta_{m+1})}, e_{(\gamma_1,\dots,\gamma_m)})_{\mathcal{F}_m} \\ &= q(j, \beta)(e_{(\beta_1,\dots,\beta_{m+1})\setminus j}, e_{(\gamma_1,\dots,\gamma_m)})_{\mathcal{F}_m} = q(j, \beta)(e_{\beta\setminus j}, e_\gamma).\end{aligned}$$

To prove (1.1), we apply the same arguments as above to reduce the situation to the case of the Fock representation.

2. We start with the following auxiliary fact.

Lemma 3.4. *In the C^*-algebra W there exist elements $\tilde{s}_j$, such that $\tilde{s}_j^* s_k = \delta_{jk} I$.*

Proof. For each $j = 1, \dots, d$, let $p_j = s_j s_j^*$ be a projection on the range of $\pi_\alpha(s_j)$. Since the C^*-algebra generated by p_j, $j = 1, \dots, d$, is finite-dimensional [8], the latter C^*-algebra, and therefore, W as well, contains $\check{p}_j = \bigvee_{k\neq j} p_k$ which is a projection on the sum of ranges of $\pi_\alpha(s_k)$, $k \neq j$.

Write $c_j = (I - \check{p}_j)s_j$. Then $c_j^* s_k = 0$, $k \neq j$, and if we show that $c_j^* s_j$ is invertible, then

$$\tilde{s}_j = (I - \check{p}_j)s_j^*(s_j^*(I - \check{p}_j)s_j)^{-1}$$

is the needed element.

So it is sufficient to prove that $c_j^* s_j = s_j^*(I - \check{p}_j)s_j$ is invertible. First notice that for any element $x = s_\mu s_\nu^*$, where μ and ν do not contain j, we have

$$s_j^* x s_j = s_j^* s_\mu s_\nu^* s_j = q(j,\mu)q(\nu,j)s_\mu s_j^* s_j s_\nu^* = q(j,\mu)q(\nu,j)x,$$

therefore, $s_j^*(I - \check{p}_j)s_j$ belongs to the finite-dimensional algebra generated by p_k, $k \neq j$, and thus its spectrum is finite. Then to prove the invertibility of $s_j^*(I - \check{p}_j)s_j$, it is enough to show that it has zero kernel, which is equivalent to $\ker(I - \check{p}_j)s_j = 0$.

Since the Fock representation of W is exact [2], any element $x \in W$ can be uniquely represented as

$$x = p_1 x_1 + \cdots + p_d x_d.$$

This means that the range of p_j is linearly independent of the span of the ranges of p_k, $k \neq j$, in particular, the ranges of p_j and $\check{p}_j$ do not intersect. This implies $\ker(I - \check{p}_j)s_j = 0$. □

Now we prove the irreducibility of π_α. For $\mu = (\mu_1, \ldots, \mu_n) \in \Lambda_n$, denote $\tilde{s}_\mu = \tilde{s}_{\mu_1} \ldots \tilde{s}_{\mu_n}$. Fix an infinite sequence $\mu = (\mu_1, \mu_2, \ldots) \in \Lambda$ and consider the operators

$$P_n(\mu) = \pi_\alpha(s_{(\mu_1,\ldots,\mu_n)}\tilde{s}^*_{(\mu_1,\ldots,\mu_n)}), \quad n \geq 1.$$

For any vector of the form e_β, one directly sees that

$$\lim_{n\to\infty} P_n(\mu)e_\beta = \delta_{\beta\mu}e_\mu,$$

i.e., the sequence $P_n(\mu)$ strongly converges to $P(\mu)$ which is a projection onto the one-dimensional space generated by e_μ.

Let C be a bounded operator commuting with all $\pi_\alpha(x)$, $x \in W$. Then C commutes with $P(\mu)$, and therefore, for any $\beta \sim \alpha$ we have $Ce_\beta = c(\beta)e_\beta$, where $c(\beta)$ is a constant. On the other hand, by the construction of $\tilde{s}_j$ given by Lemma 3.4, for each $\beta, \gamma \sim \alpha$ there exist finite multiindices μ, ν, such that $s_\gamma = s_\mu \tilde{s}_\nu^* s_\beta$, so that $\pi_\alpha(s_\mu \tilde{s}_\nu^*)e_\beta = e_\gamma$. Since C commutes with $\pi_\alpha(s_\mu \tilde{s}_\nu^*)$, we have

$$c(\beta)e_\gamma = c(\beta)\pi_\alpha(s_\mu\tilde{s}_\nu^*)e_\beta = \pi_\alpha(s_\mu\tilde{s}_\nu^*)Ce_\beta = C\pi_\alpha(s_\mu\tilde{s}_\nu^*)e_\beta = Ce_\gamma = c(\gamma)e_\gamma,$$

i.e., $c(\beta) = c(\gamma)$ for all $\beta, \gamma \sim \alpha$. Therefore, C is a scalar operator, and by the Schur lemma, π_α is irreducible.

3. Using the arguments above, if α' is not equivalent to α, then

$$\lim_{n\to\infty} P_n(\alpha')e_\beta = 0, \quad \beta \sim \alpha,$$

therefore, $P_n(\alpha')$ strongly converges to zero in H_α, but, in $H_{\alpha'}$, it strongly converges to a nonzero projection $P(\alpha')$.

4. Assume that there exists $\Omega \in H_\alpha$, for which $\pi_\alpha(s_j)^*\Omega = 0$, $j = 1, \ldots, d$. For each $\beta \sim \alpha$ we have

$$(\Omega, e_\beta) = (\Omega, \pi_\alpha(s_{\beta_1})e_{\sigma(\beta)}) = (\pi_\alpha(s_{\beta_1})^*\Omega, e_{\sigma(\beta)}) = 0.$$

Since vectors e_β, $\beta \sim \alpha$, form a total set in H_α, we get $\Omega = 0$. □

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