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FIXED POINT THEOREM FOR ψ-GERAGHTY CONTRACTION TYPE MAPPINGS IN B-METRIC SPACES WITH APPLICATION

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Abstract. In this paper, we introduce a new class of contractive mappings, called generalized ψ-Geraghty contractions, in the framework of b-complete metric spaces. We establish a unique fixed-point theorem that extends existing results in fixed-point theory. An illustrative example with a graphical representation demonstrates the validity of our findings. Furthermore, we apply the main result to an integral equation, highlighting its effectiveness in ensuring the existence and uniqueness of solutions. This work underscores the theoretical significance and practical applicability of generalized ψ-Geraghty contractions in mathematics, physics, and engineering.

1. Introduction and Preliminaries

In 1922, S. Banach [1] presented a fundamental result in fixed point theory. Since then, this area has been extensively studied, further developed, and generalized by many researchers in various spaces. In 1989, Bakhtin [6] introduced b-metric spaces as a generalization of metric spaces. The works of Bakhtin [6], Bourbaki [2], and Czerwik [3] were among the earliest contributions to extending fixed point theory within the framework of b-metric spaces. Moreover, several authors have established significant fixed point theorems in b-metric spaces, including the contributions of Berinde [4] and Vulpe [5].

One of the significant generalizations was introduced by M.A. Geraghty [7] in 1973, where the classical contractive condition was relaxed by allowing the contraction factor to depend on the distance between points. This led to the development of what is now known as the Geraghty contraction, which has been applied in various mathematical contexts. Over the years, researchers have extended this concept to more general spaces, such as b-metric spaces [8], ordered b-metric spaces with rational contractive conditions [9], partial metric spaces [11], and $\alpha - \psi$- type contractions [12]. These generalizations have significantly broadened the applicability of fixed point results in both theoretical and practical problems.

Definition 1. [6]. Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to R_+$ is said to be b-metric if and only if $\forall\ x, y, z \in X$ the following conditions are satisfied:

(1) $d(x, y) = 0$ if and only if $x = y$,
(2) $d(x, y) = d(y, x)$,
(3) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

A triplet (X, d, s), is called a b-metric space with coefficient 's'.

Michael A. Geraghty [7] introduced the Geraghty contraction in 1973, he used it for the existence and uniqueness of mappings in any complete metric spaces.

Definition 2. [7] A Geraghty contraction is a mapping that satisfies the inequality

$$(d(Tx, Ty)) \leq \beta(d(x, y))(d(x, y)),\ x, y \in X$$

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where $\beta \in S$ Let B denote the set of all functions $\beta : [0, \infty) \to [0, \frac{1}{s})$ which satisfy the condition

$$\limsup_{n\to\infty} \beta(t_n) = \frac{1}{s} \implies t_n \to 0 \text{ as } n \to \infty.$$

Karapinar [12] introduced the concept of $\alpha - \psi$- Geraghty contraction-type mappings in complete metric spaces.

Definition 3. [12] Let Ψ denote the class of the functions $\psi : [0, \infty) \to [0, \infty)$ satisfying the following conditions:

(1) ψ is nondecreasing,
(2) ψ is continuous,
(3) $\psi(t) = 0$ if and only if $t = 0$,
(4) ψ is subadditive, that is $\psi(s + t) \leq \psi(s) + \psi(t)$.

In 2019 Hamid Faraji et al. [8] introduced common fixed point of Geraghty type contractive mappings defined in complete b-metric spaces.

Let B denote the set of all functions $\beta : [0, \infty) \to [0, \frac{1}{s})$ which satisfy the condition:

$$\limsup_{n\to\infty} \beta(t_n) = \frac{1}{s} \implies t_n \to 0 \text{ as } n \to \infty.$$

The objective of this work is to introduce generalized ψ-Geraghty contractive mappings and prove fixed point theorems in ψ-Geraghty contractive mappings in b-complete b-metric spaces. Our results generalize or improve many recent fixed point theorems in the literature. We provide an example to validate our result.

2. Main Result

In this section, we establish the fixed-point theorem in b-complete b-metric space based on the ψ-Geraghty contractive mappings.

Theorem 1. *Let (X, d) be a b-complete b-metric space with parameter $s \geq 1$. Let $T : X \to X$ be a self-mapping satisfying,*

$$\psi(d(Tx, Ty)) \leq \beta(\psi(M(x, y)))\psi(M(x, y)), \;\; x, y \in X \tag{2.1}$$

where

$$M(x, y) = \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{1}{2s}\left(d(x, Ty) + d(y, Tx)\right)\right\},$$

and $\beta \in \boldsymbol{B}$. Then T has a unique fixed point.

Proof. Let $x_0 \in X$ be arbitrary. Consider the sequence $\{x_n\}$, where $x_n = Tx_{n-1} = T^n x_0, \quad n \in \mathbb{N}$. If there exists $n \in \mathbb{N}$ such that $x_{n+1} = x_n$, then x_n is a fixed point of T and the proof is finished. Otherwise, we have $d(x_{n+1}, x_n) > 0$ for all $n \in \mathbb{N}$. By inequality (2.1) , for all $n \in \mathbb{N}$ we have,

$$\psi(d(x_n, x_{n+1})) = \psi(d(Tx_{n-1}, Tx_n)) \leq \beta(\psi(M(x_{n-1}, x_n)))\psi(M(x_{n-1}, x_n)), \tag{2.2}$$

where,

$$\begin{aligned}
&M(x_{n-1},x_n)\\
&=\max\left\{d(x_{n-1},x_n),d(x_{n-1},Tx_{n-1}),d(x_n,Tx_n),\frac{d(x_{n-1},Tx_n)+d(x_n,Tx_{n-1})}{2s}\right\}\\
&=\max\left\{d(x_{n-1},x_n),d(x_{n-1},x_n),d(x_n,x_{n+1}),\frac{d(x_{n-1},x_{n+1})+d(x_n,x_n)}{2s}\right\}\\
&=\max\left\{d(x_{n-1},x_n),d(x_n,x_{n+1}),\frac{s(d(x_{n-1},x_n)+d(x_n,x_{n+1}))}{2s}\right\}\\
&=\max\left\{d(x_{n-1},x_n),d(x_n,x_{n+1})\right\}.
\end{aligned}$$

If $(d(x_{n-1},x_n)\le d(x_n,x_{n+1})$, then $M(x_{n-1},x_n)=d(x_n,x_{n+1})$. From condition(2.2), we obtain,

$$\begin{aligned}
d(x_n,x_{n+1})&\le\beta(\psi(M(x_{n-1},x_n))(\psi(M(x_{n-1},x_n))\\
&\le\frac{1}{s}(\psi(d(x_n,x_{n+1})),\quad n\in\mathbb{N}\\
&<\frac{1}{s}(d(x_n,x_{n+1}))\\
&<d(x_n,x_{n+1}).
\end{aligned}$$

This is a contradiction. Thus, we have,

$$M(x_{n-1},x_n)=d(x_n,x_{n-1}),$$

then, from inequality (2.2), We obtain,

$$\begin{aligned}
\psi(d(x_n,x_{x+1}))&\le\beta(\psi(M(x_{n-1},x_n)))\psi(d(x_{n-1},x_n))\\
&<\frac{1}{s}(d(x_{n-1},x_n)),\quad n\in\mathbb{N}\\
&<d(x_{n-1},x_n),\quad n\in\mathbb{N}.
\end{aligned}\tag{2.3}$$

Since ψ is non-decreasing, we have $(d(x_n,x_{n+1})\le d(x_{n-1},x_n)$ for all $n\in\mathbb{N}$. Hence, we deduce that sequence $\{d(x_{n-1},x_n)\}$ is a decreasing sequence. Therefore, there exists $r\ge 0$ such that $\lim_{n\to\infty} d(x_{n-1},x_n)=r$, we claim that $r=0$, suppose that $r>0$, then from inequality (2.3), we have,

$$\psi(d(x_n,x_{n+1}))\le\beta(\psi(M(x_{n-1},x_n)))\psi(d(x_{n-1},x_n)),$$

so,

$$\frac{\psi(d(x_n,x_{n+1}))}{\psi(d(x_{n-1},x_n))}\le\beta(\psi(Md(x_{n-1},x_n)))\le\frac{1}{s}.$$

This implies that $\lim_{n\to\infty}\beta(\psi(Md(x_{n-1},x_n)))=\frac{1}{s}$, since $\beta\in\mathbf{B}$, we have $\lim_{n\to\infty}\psi(M(x_{n-1},x_n))=0$, which yields

$$r=\lim_{n\to\infty}d(x_{n-1},x_n)=0,$$

which is a contradiction, that is, $r=0$. Now we show that $\{x_n\}$ is a b-Cauchy sequence. Suppose, on the contrary that $\{x_n\}$ is not a b-Cauchy sequence. Then there exists $\epsilon>0$ for which we can find sub sequences $\{x_{m(k)}\}$ and $\{x_{n(k)}\}$ of $\{x_n\}$ such that $n(k)$ is the smallest index for which $n(k)>m(k)>k$,

$$d(x_{m(k)},x_{n(k)})\ge\epsilon,\tag{2.4}$$

and

$$d(x_{m(k)},x_{n(k-1)})<\epsilon,\tag{2.5}$$

from equation (2.5) and using the b-triangular inequality, we have,

$$\epsilon \leq d(x_{m(k)}, x_{n(k)}) \leq s\left(d(x_{m(k)}, x_{m(k)+1}) + d(x_{m(k)+1}, x_{n(k)})\right).$$

Letting $k \to \infty$, we have,

$$\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = \epsilon,$$

since $|\{d(x_{m(x)}, x_{n(k)-1}) - d(x_{m(x)}, x_{n(k)})\}| \leq d(x_{m(x)}, x_{m(k)-1})$, we have

$$\lim_{k\to\infty} d(x_{m(x)}, x_{m(k)-1}) = \epsilon.$$

Then, we get,

$$\frac{\epsilon}{s} \leq \limsup_{k\to\infty} d(x_{m(x)+1}, x_{n(k)}), \tag{2.6}$$

therefore,

$$\begin{aligned}\psi(d(x_{m(k)}, x_{n(k)}) &= \psi(d(Ax_{m(k)-1}, Ax_{n(k)})\\ &\leq \beta(\psi(M(x_{m(k)-1}, x_{n(k)-1})(\psi(M(x_{m(k)-1}, x_{n(k)-1}),\end{aligned}$$

where,

$$\begin{aligned}&\limsup_{k\to\infty} M(x_{m(x)}, x_{n(k)-1}))\\ &= \limsup_{k\to\infty} \max\{d(x_{m(k)}, x_{n(k)-1})), d(x_{m(k)}, Tx_{m(k)}),\\ &\quad d(x_{n(k)-1}, Tx_{n(k)-1}), \frac{\{d(x_{m(k)}, Tx_{n(k)-1}) + d(x_{n(k)-1}, Tx_{m(k)})}{2s}\}\\ &= \limsup_{k\to\infty} \max\{d(x_{m(k)}, x_{n(k)-1}), d(x_{m(k)}, x_{m(k)+1}),\\ &\quad d(x_{n(k)-1}, x_{n(k)}), \frac{\{d(x_{m(k)}, x_{n(k)}) + d(x_{n(k)-1}, x_{m(k)+1})}{2s}\}\\ &\leq \limsup_{k\to\infty} \max\{d(x_{m(k)}, x_{n(k)-1}), d(x_{m(k)}, x_{m(k)+1}),\\ &\quad d(x_{n(k)-1}, x_{n(k)}), \frac{sd(x_{m(k)}, x_{n(k)-1}) + sd(x_{n(k)}, x_{n(k)-1})}{2s}\\ &\quad + \frac{sd(x_{m(k)-1}, x_{m(k)}) + sd(x_{m(k)}, x_{m(k)+1})}{2s}\}\\ &\leq \epsilon.\end{aligned}$$

Therefore, $\lim_{k\to\infty} M(x_{m(k)}, x_{n(k)-1}) = \epsilon$,

$$\frac{1}{s} = \lim_{k\to\infty} \frac{\psi(d(x_{m(n)}, x_n))}{\psi(d(M(x_{n(k)}, x_{m(k)-1}))} \leq \lim_{k\to\infty} \beta(\psi(M(x_{m(k)-1}, x_{n(k)-1}))) \leq \epsilon,$$

$$\limsup_{k\to\infty} \beta(\psi(M(x_{m(k)}, x_{n(k)-1}))), \text{which implies} \lim_{k\to\infty} \psi(M(x_{m(k)-1}, x_{n(k)-1})) = \frac{1}{s}.$$

Then, $\frac{1}{s} \leq \limsup_{k\to\infty} \beta(M(x_{m(k)}, x_{n(k)-1})) \leq \frac{1}{s}$. Since $\beta \in \mathbf{B}$,
so $M(x_{m(k)}, x_{n(k)-1}) \to 0$, as a result, $d(x_{m(k)}, x_{n(k)-1}) \to 0$. From inequality (2.4) and using the b-triangular inequality, we have,

$$\epsilon \leq d(x_m(k), x_n(k)) \leq (s(d(x_m(k), x_n(k) - 1) + (d(x_n(k) - 1, x_n(k))).$$

Therefore, $\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = 0$. Hence $\epsilon = 0$. This contradicts inequality (2.4). Hence sequence $\{x_n\}$ is a b-Cauchy sequence. The completeness of X implies that there exists $u \in X$ such that $x_n \to u$. We showed that u is a fixed point of T. By b-triangular inequality and inequality (2.1), we have,

$$\begin{aligned}\psi(d(u,Tu) &\leq s(d(u,Tx_n)+d(Ax_n,Tu))\\ &\leq s(d(u,Tx_n)+s\beta(\psi M(x_n,u))\psi(M(x_n,u)\end{aligned}$$

letting $n \to \infty$ in the above inequality, we obtain,

$$\psi(d(u,Tu)) \leq \limsup_{n\to\infty} d(u,x_{n+1}) + s\limsup_{n\to\infty}\beta(\psi(M(x_n,u)))\limsup_{n\to\infty}\psi(M(x_n,u)) \quad (2.7)$$

where,

$$\begin{aligned}&\limsup_{n\to\infty} M(x_n,u)\\ &=\limsup_{n\to\infty}\max\{d(x_n,u),d(x_n,Tx_n),d(u,T_u),\frac{1}{2s}(d(x_n,Tu)+d(u,Tx_n))\}\\ &\leq \limsup_{n\to\infty}\max\{d(x_n,u),d(x_n,Tx_{n+1}),d(u,T_u),\frac{1}{2s}(sd(x_n,u)\\ &\qquad\qquad + sd(u,Tu)+d(u,x_{n+1}))\}\\ &\leq d(u,Tu).\end{aligned}$$

Hence, from inequality (2.7), we have,

$$\psi(d(u,Tu)) \leq s\limsup\beta(\psi(M(x_n,u))\psi(d(u,Tu))$$

. Consequently, $\frac{1}{s} \leq \limsup_{n\to\infty}\beta(Md(x_n,u)) \leq \frac{1}{s}$. Since $\beta \in \mathbf{B}$, we concluded $\lim_{n\to\infty} M(x_n,u)=0$. Therefore, $Tu=u$, we have to prove that the fixed point $u\in X$ is unique, suppose that there is $v\neq u$ in X such that $Tv=v$. From inequality (2.1), we get,

$$\psi(d(u,v))=\psi(d(Au,Av)\leq\beta\Psi(M(u,v))\Psi(M(u,v)),$$

where,

$$\begin{aligned}M(u,v)&=\max\{d(u,v),d(u,T_u),d(v,T_v),\frac{1}{2s}(d(u,T_v)+d(v,T_u)\}\\ &\leq d(u,v).\end{aligned}$$

Therefore, we have $d(u,v)<\frac{1}{s}d(u,v)$. Then $u=v$, which is a contradiction. □

In this section, we give an example to validate our result.

Example 1. Let $X=[0,\infty)$ and $d: X\times X\to[0,1]$ be defined by $d(x,y)=|x-y|^2$ for all $x,y\in[0,\infty)$ and (x,d) b-complete b-metric space with parameter $s=2$. Let $Tx=\frac{x}{5}$ afor all $x\in X$ and $\beta=\frac{1}{1+2t}$ or $\psi(t)=\frac{t}{2}$. $s\geq 1$. Then T has a unique fixed point $x^*\in X$.
Solution: We consider the following three cases.
Case-1: When $x,y\in[0,1]$ and $x<y$

$$\psi(d(Tx,Ty))=\beta(\psi(M(x,y))\psi(M(x,y)),$$

where,

$$\begin{aligned}M(x,y)&=\max\left\{d(x,y),d(x,Tx),d(y,Ty),\frac{1}{2s}\left(d(x,Ty)+d(y,Tx)\right)\right\},\\ &=\max\left\{|x-y|^2,|x-Ax|^2,|y-Ay|^2,\frac{1}{2s}\left(|x-Ay|^2+|y-Ax|^2\right)\right\},\\ &=\max\left\{|x-y|^2,|x-x|^2,|y-y|^2,\frac{1}{2s}\left(|x-y|^2+|y-x|^2\right)\right\},\\ &=|x-y|^2.\end{aligned}$$

so,

$$\begin{aligned}\psi(d(Tx,Ty)) &= \frac{1}{2}(|\frac{x}{5}-\frac{y}{5}|^2)\\ &= \frac{1}{50}(|x-y|^2)\\ &\leq \beta(\psi(|x-y|^2))\psi(|x-y|^2),\\ &= \beta(\frac{|x-y|^2}{2})\frac{|x-y|^2}{2}\\ &= (\frac{|x-y|^2}{4}\frac{1}{1+2|x-y|^2}) \end{aligned} \tag{2.8}$$

x	y	**L.H.S** $\left(\frac{1}{50}\vert x-y\vert^2\right)$	**R.H.S** $\left(\frac{\vert x-y\vert^2}{4}\cdot\frac{1}{1+2\vert x-y\vert^2}\right)$
0	0.5	0.005	0.0167
0.2	0.7	0.005	0.0145
0.5	1.0	0.005	0.0125
⋮	⋮	⋮	⋮

TABLE 1. Variation of L.H.S and R.H.S for specific values of x and y in the range $[0,1]$, with $x < y$.

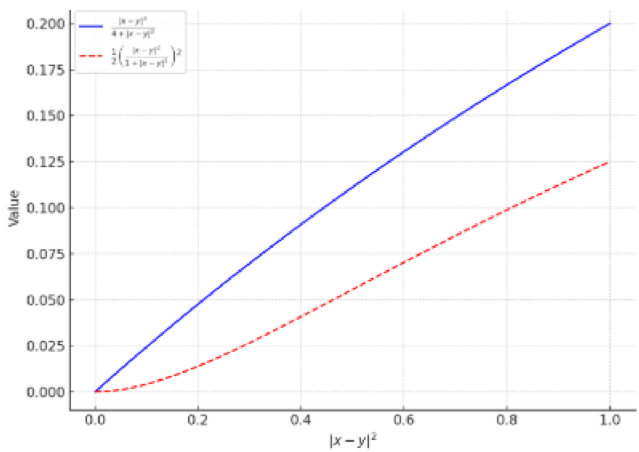

FIGURE 1. Variation of Left hand Side (L.H.S.) and Right-hand Side (R.H.S) when $x, y \in [0,1]$ and $x < y$

Case-2: When $x, y \in [0, 1]$ and $x > y$. We analyze and interpret the data using both the graph and comparison table.

x	y	**L.H.S** $\left(\frac{1}{50}\lvert x-y\rvert^2\right)$	**R.H.S** $\left(\frac{\lvert x-y\rvert^2}{4} \cdot \frac{1}{1+2\lvert x-y\rvert^2}\right)$
0.5	0.4	0.0002	0.00083
0.7	0.3	0.0032	0.0121
0.9	0.1	0.0128	0.0388
$\vdots$	$\vdots$	$\vdots$	$\vdots$

TABLE 2. Variation of L.H.S and R.H.S for specific values of x and y in the range $[0, 1]$, with $x > y$.

From case 1 and case 2 for $x < y$ (or $x > y$, since the expressions are symmetric xy) the nonzero values of xy both expressions are positive. L.H.S grows linearly with xy^2, while R.H.S. grows more slowly due to the additional denominator factor $1 + |x - y|^2$. We conclude that for $x \neq y$, R.H.S is generally larger than L.H.S. as $|x - y|$ grows, this difference increases.

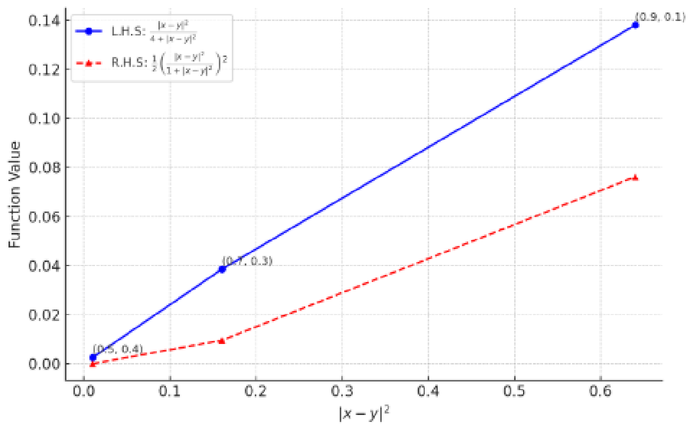

FIGURE 2. Variation of Left hand Side (L.H.S.) and Right-hand Side (R.H.S) when $x, y \in [0, 1]$ and $x > y$

Case-3: When $x, y \in [0, 1]$ and $x = y$. In this cases, the expressions are equal and yield a value of zero. There is no need for a comparison table in this case, as the result is always 0 for both sides. By plotting these functions over the interval $[0, 1]$, we can observe where the two curves meet. The points of intersection correspond to values of xy for which the two expressions are equal. These points reveal potential fixed points, including the trivial fixed point at $x = y$ (where $|xy| = 0$), as well as any non-trivial fixed points that might exist within this range.
Therefore, the conditions of Theorem 1 are satisfied.

3. Application

In this section, we study the existence of solutions for nonlinear integral equations as an application to the fixed point theorems proved in the previous section. Let $X = C[0, l]$

be the set of all real continuous functions on $[0, l]$ and $d : X \times X \to [0, \infty)$ be defined by:

$$d(u, v) = \max_{0 \le t \le l} |u(t) - v(t)|^2, u, v \in X.$$

Obviously, (X, d) is a complete b-metric space with parameter $s = 2$. First, consider the integral equation:

$$u(t) = h(t) + \int_0^l G(t, s)k(t, s, u(s))ds, \tag{3.9}$$

where $l > 0$ and $h : [0, l] \to R$, $G : [0, l] \times [0, l] \times \to R$ and $k : [0, l] \times [0, l] \times R \to R$ are continuous functions.

Theorem 2. *Suppose that the following hypotheses hold:*
(1) *For all $t, s \in [0, l]$ and $u, v \in X$, we have,*

$$|k(t, s, u(s)) - k(t, s, v(s))| \le \frac{1}{1 + 2M(u, v)} M(u, v).$$

(2) *For all $t, s \in [0, l]$, we have,*

$$\max \int_0^l G(t, s)^2 ds \le \frac{1}{l}.$$

Then, the integral equation (3.9) has a unique solution $u \in X$.

Proof. Let $A : X \to X$ be a mapping defined by:

$$Au(t) = h(t) + \int_0^1 G(t, s)k(t, s, u(s))ds, \quad u \in X, t, s \in [0, l].$$

From inequality (2.1) and inequality (2.2), we can write:

$$\begin{aligned}
&\psi(d(Au, Av)) \\
&= \max_{t \in [0,l]} |Au(t) - Av(t)|^2 \\
&= \max_{t \in [0,l]} \{|h(t) + \int_0^l G(t, s)k(t, s, u(s))ds - h(t) - \int_0^l G(t, s)k(t, s, v(s))ds|^2\} \\
&= \max_{t \in [0,l]} |\int_0^l G(t, s)(k(t, s, u(s)) - k(t, s, v(s)))ds|^2 \\
&\le \max_{t \in [0,l]} \int_0^l G(t, s)^2 ds \int_0^l |(k(t, s, u(s)) - k(t, s, v(s)))|^2 ds \\
&\le \frac{1}{l} \int_0^l |\frac{1}{1 + 2M(u, v)} M(u, v)| ds \\
&\le \frac{M(u, v)}{1 + 2M(u, v)}.
\end{aligned}$$

so we get,

$$\psi(d(Au, Av)) \le \beta(\psi(M(u, v)))\psi(M(u, v)).$$

Thus, all conditions in Theorem 2 for $\psi(t) = t$, $t > 0$ where $\beta(t) = \frac{1}{1+2t}$ are satisfied and hence T has a fixed point $x = 0$. □

4. Conclusion

From our investigations we conclude that the ψ-contractive defined on a b-complete b-metric space satisfying ψ-Geraghty contractive mappings and have a unique common fixed point. Our investigations and results obtained were supported by the suitable example with graphs which provides new path for researchers in the concerned field.

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